

Math 2250-3
Friday August 27, 2004
First order differential equations - notes for % 1.1-1.3

HW for Wednesday September 1: (bold means hand in, others optional. You already got this assignment on Wednesday, and it's also posted at our home page.)

% 1.1: 3, 4, **7**, 15, **16**, 19, **20**, 27, **30**, **34**, **36**, **46**

% 1.2: 5, **7**, **10**, **14**, **20**, **25**, **26**, **36**, **44**

% 1.3: 3, **6**, **8**, 11, **12**, **14**, **21**, **25**, 32, 33

Course home page: <http://www.math.utah.edu/~korevaar/2250fall04>

Tuesday problem sessions: 7:30-8:20 a.m. in EMCB 101; 10:45-11:35 a.m. in LS 101;
12:55-1:45 p.m. in EMCB 105; 5-5:50 p.m. in LCB 219.

Maple and Math lab introductions: All are in LCB 115. Tuesday August 31: 11:50-12:40,
Wednesday September 1: 2-2:50, Friday September 3, 10:45-11:35, 11:50-12:40.

What is a differential equation?

What is the order of a differential equation?

How do you transform a sentence about rates of change for a function into a differential equation? Can you do % 1.1 #34?

How do you check whether a function solves a differential equation? Can you do % 1.1 #4? Or see example 2 below.

A **First order differential equation** is any equation which can be written as

$$\left[F\left(x, y, \frac{dy}{dx}\right) = 0 \right.$$

For example,

$$\left[y + \left[\frac{dy}{dx}\right]^2 = 1 \right.$$

is a differential equation. Often we can rewrite a general D.E. in the more convenient form

$$\left[\frac{dy}{dx} = f(x, y) \right.$$

Can you do so for the example above? No matter how we write the differential equation, the goal is to understand the functions $y(x)$ which make it true. Sometimes we want the particular solution $y(x)$ which satisfies an **initial condition**

$$\left[y(x_0) = y_0 \right.$$

Then the corresponding problem is called an **initial value problem**. The reason for the name "initial condition" is that our independent variable will often be time "t" rather than x, and the initial condition will be the value of the solution function at time $t=0$, for example an initial population in a population growth problem.

In section 1.2 the book focuses on the easiest kind first order differential equations to solve, namely ones of the form

$$\left[\frac{dy}{dx} = f(x) \right]$$

Notice the unknown function $y(x)$ does not appear on the right hand side of this D.E so you are simply looking for antiderivatives of $f(x)$ with respect to x :

$$\left[y = \int f(x) dx + C \right]$$

You got good at antidifferentiation in Calculus!!!! Basic gravitational attraction problems in physics are of this form, when "t" is the variable. Sometimes more complicated problems also reduce to antidifferentiation.

Example 1. Solve this initial value problem, and then graph the solution function:

$$\left[\begin{aligned} \frac{dy}{dx} &= x - 3 \\ y(1) &= 2 \end{aligned} \right]$$

Here's how Maple would solve the problem:

```
[ > with(DEtools): #load differential equations package
[ > dsolve({diff(y(x),x)=x-3,y(1)=2},y(x));
[
[ >
```

$$y(x) = \frac{1}{2}x^2 - 3x + \frac{9}{2}$$

Maple is a word and math processing package (think Microsoft word that does math). You can buy Maple from the bookstore for around \$100, or use it on campus computer systems, where it already exists.

Example 2. Solve the initial value problem:

$$\begin{cases} \frac{dy}{dx} = y - x \\ y(0) = 0 \end{cases}$$

Hint: You CANNOT solve this problem by pure antidifferentiation since you can't antidifferentiate the $y(x)$ on the right side if you don't know it yet. We WILL learn an algorithm for solving this sort of DE in section 1.4. For today you may use the fact that the general solution to this differential equation is given by

$$y(x) = x + 1 + C e^x$$

Step 1: verify that $y(x)$ does solve the DE.

Step 2: find a value of C in the general solution to solve the IVP.

Step 3: graph the solution function.

Slope Fields

And now for something which seems completely different (but isn't). The geometric interpretation of first order differential equations is connected with slope fields. (The use of the word "field" is analogous to how it's used with the prefixes "force", "electric", "magnetic", "wheat".) Slope fields will be discussed in more detail on Monday, and we introduce them now. The differential equation

$$\frac{dy}{dx} = f(x, y)$$

is telling us that the slope of the solution graph at a point $(x, y(x))$, is given by the function $f(x, y)$. We can use this fact to approximate solutions, especially when exact formulas cannot be found: By hand or computer we can draw a picture of the **slope field** determined by f , i.e. at each point (x, y) assign a slope using the formula $f(x, y)$. The natural way to indicate these slopes is with small line segments, see below. Solution graphs to the differential equation will be tangent to these slope fields. If there is an initial condition

$$y(x_0) = y_0$$

then the solution graph will pass through the point

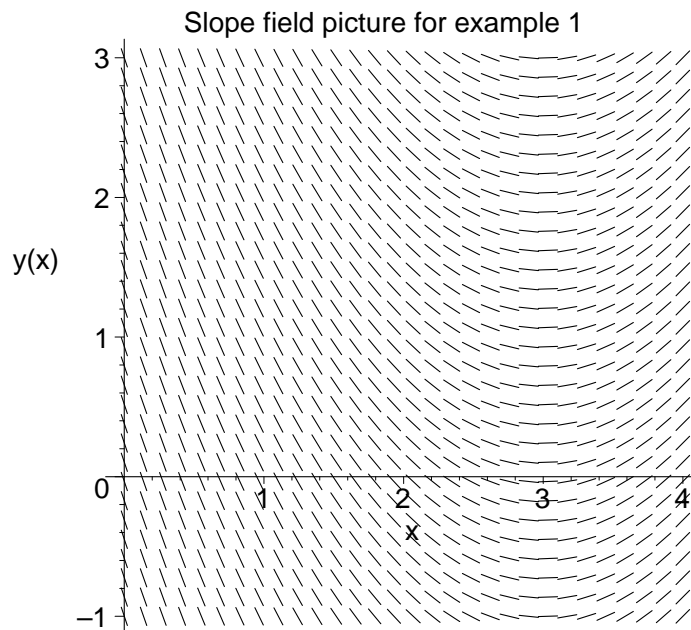
$$[x_0, y_0]$$

Example 1. The initial value problem:

$$\frac{dy}{dx} = x - 3$$

$$y(1) = 2$$

Picture of slope field:



Example 2. The initial value problem:

```
> dy/dx = y-x;  
y(0)=0;
```

$$\frac{dy}{dx} = y - x$$

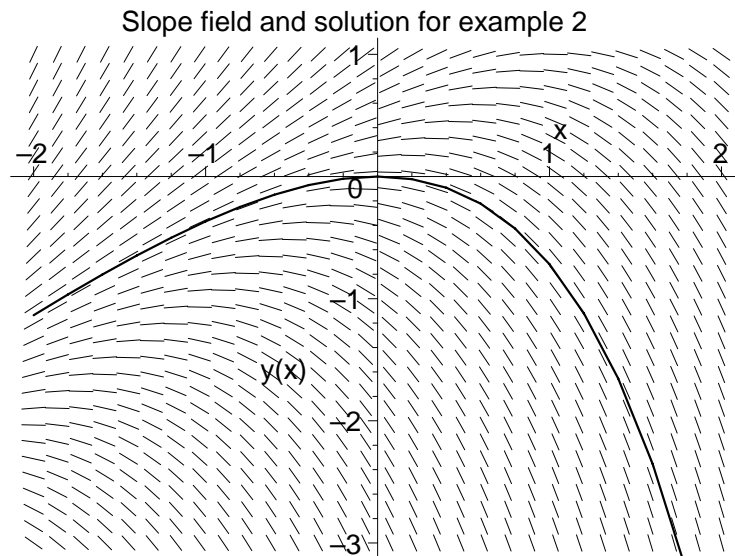
$$y(0) = 0$$

Geometric picture: (and the Maple commands that generated it.)

```
> deqtn:=diff(y(x),x)=y(x)-x; #this is example 2  
dsolve({deqtn,y(0)=0},y(x)); #Maple solution  
DEplot(deqtn,y(x),x=-2..2,[y(0)=0],y=-3..1,  
arrows=line,color=black,linecolor=black,  
dirgrid=[30,30],  
title='Slope field and solution for example 2');  
#make a picture!
```

$$deqtn := \frac{d}{dx} y(x) = y(x) - x$$

$$y(x) = x + 1 - e^x$$



```
>
```

Some real examples!
Example 2 page 12-13:

Example 4, page 15-16: