

Math 1210-001
Tuesday Mar 8
WEB L110

Section 3.4 Optimization problems, continued.

- Finish Snell's law example in yesterday's notes, then work on today's examples.

Exercise 1) Consider open-top right-circular cylinders of fixed volume V . How should the radius r of the base and the height h be related, in order to minimize the total surface area of the cylinder - counting the sides, the top and the bottom? Are these dimensions close to those of actual planters? How could actual material costs differ from our model?



There are steps to organize your work, analogous to how we approached related rates:

- 1) Draw a picture and assign variables to the quantities that can change.
- 2) Write a formula for the function Q you are trying to maximize or minimize. This function is called the "objective function".
- 3) If the objective function involves more than one variable, use the conditions ("constraints") of the problem to eliminate all but one of these variables, i.e. so that you can (re)write Q as a function of one variable.
- 4) Find the critical points (end points, stationary points, singular points)
- 5) Either use critical point theory from section 3.1 or first and second derivative information from sections 3.2, 3.3 to determine the maximum or minimum value you desire, and the corresponding optimal configuration.

Exercise 2) (Example 4, page 169) Suppose a fish swims upstream with velocity v relative to the water, and that the river current has velocity $-v_c$ relative to the shoreline (in the opposite direction the fish is swimming). Assume the energy expended by the fish to travel a distance d up the river is proportional to the time it takes to travel this distance and the cube of the fish's velocity relative to the water.

What is the velocity v that minimizes the energy expended in swimming this distance? (Do fish actually swim at this speed?)

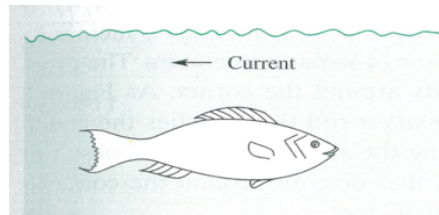


Figure 5

Exercise 3) (Exercise 36, page 175). A rectangle is to be inscribed in a semicircle of radius r as shown below. What are the dimensions of the rectangle that maximize its area?

