Math 1210-001
Wednesday Mar 30
WEB L112

- Lab or WebWork problems?
- For most of today we will use Tuesday's notes about area and summation notation, section 4.1
- The discussion there leads to the definition of the "definite integral" (or "Riemann integral") in section 4.2. We may have time to discuss this a bit today. This week's lab and WebWork assignments will follow up.

The definite integral:

A) Partition: Let $[a, b]$ be a closed interval. A partition "P" of $[a, b]$ is a decomposition of $[a, b]$ into some counting number n of subintervals,

$$P: \ a = x_0 < x_1 < x_2 < .... < x_{n-1} < x_n = b.$$

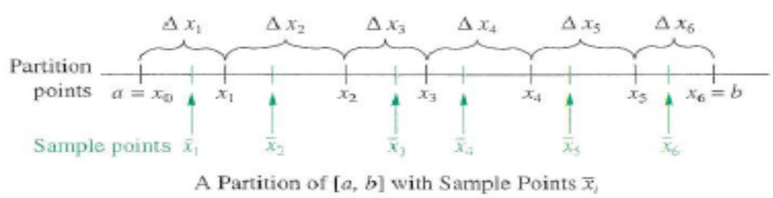
In other words, for $1 \leq i \leq n$, the i^{th} subinterval is

$$[x_{i-1}, x_i].$$

We call the width of the i^{th} subinterval Δx_i:

$$\Delta x_i = x_i - x_{i-1}.$$

We pick specified sample points $\underline{x_i}$ in each subinterval, $x_{i-1} \leq \underline{x_i} \leq x_i$. (The text uses overlines rather than underlines for the sample points.) Often the sample points are left endpoints or right endpoints of the subintervals. Here is a picture of a partition of an interval $[a, b]$ into 6 subintervals, along with choices of sample points:



A Partition of $[a, b]$ with Sample Points $\bar{x}_i$

B) The Riemann Sum R_P for the partition P and choice of sample points as above is the sum

$$R_P = \sum_{i=1}^{n} f(\underline{x_i}) \Delta x_i.$$

If the function f is positive on the interval $[a, b]$ then the Riemann sum can be thought of as an approximation for the area between the graph of f and the horizontal axis:

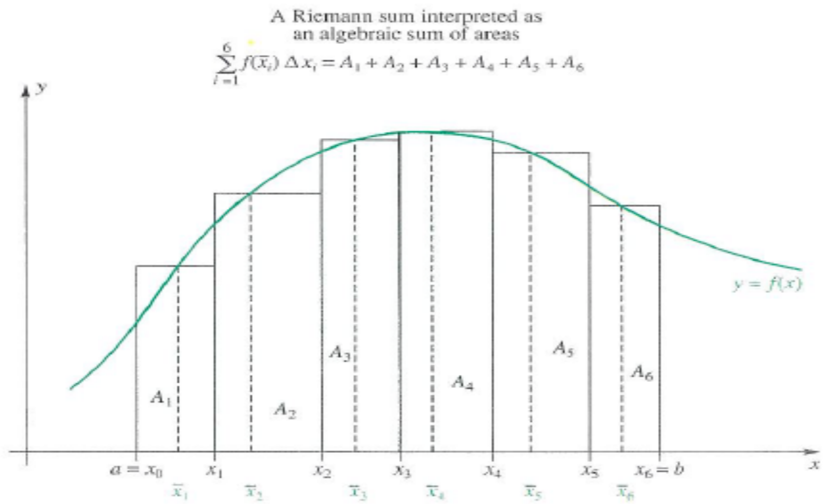


Figure 3

If the function f changes sign on the interval $[a, b]$ then the Riemann sum approximates the net signed area; on subintervals where $f(x_i) < 0$ the contribution $f(x_i)\Delta x_i$ is negative, and is the opposite of the rectangle area between the graph and the horizontal axis:

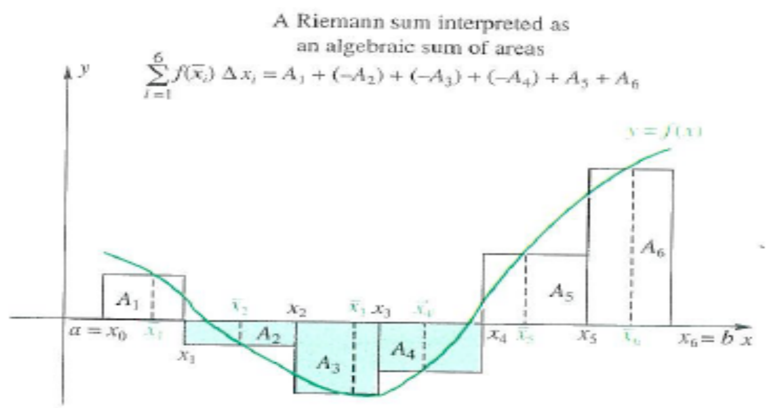


Figure 5

C) The "norm" of a partition P is the largest of the subinterval widths Δx_i. We denote the norm of the partition with the symbol $\|P\|$.

D) The definite integral of f, on the interval $[a, b]$ is denoted by $\int_a^b f(x)\,dx$ and is defined as the limit:

$$\int_a^b f(x)\,dx := \lim_{\|P\|\to 0} R_p = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(\underline{x}_i)\Delta x_i.$$

This limit always exists if f is continuous on $[a, b]$, and represents the exact net "signed area" between the graph of f and the horizontal axis.

Example In Tuesday's notes (that we cover on Wednesday), we showed that

$$\int_0^1 x^2\,dx = \frac{1}{3}.$$