Math 1210-001
Monday Mar 28
WEB L112

- Complete Friday's notes, which begin section 3.9: introduction to differential equations.
- Today's notes finish section 3.9.
- On Tuesday we begin 4.1-4.2: "The definite integral" - these & derivatives = the key objects in Calc 1.
- Fri, Mon, Tues: 4.3-4.4: The two fundamental theorems of calculus - connect definite integrals and antidifferentiation.
- Midterm 3 is next week Friday.

space for warm-up antidifferentiation problem:

- The slope field picture for a differential equation

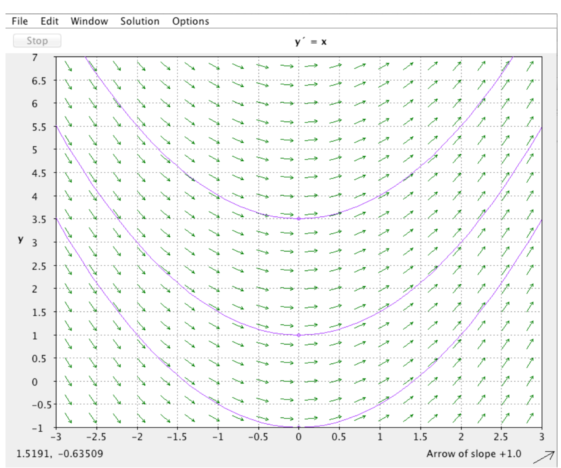
$$\frac{dy}{dx} = m(x, y)$$

is constructed by plotting lots of line segements with varying slope $m(x, y)$ at points (x, y) in the plane. Graphs of solutions $y(x)$ will be tangent to the constructed slope field.

<u>Example</u> On Friday we solved the differential equation initial value problem

$$\frac{dy}{dx} = x$$
$$y(0) = 1$$

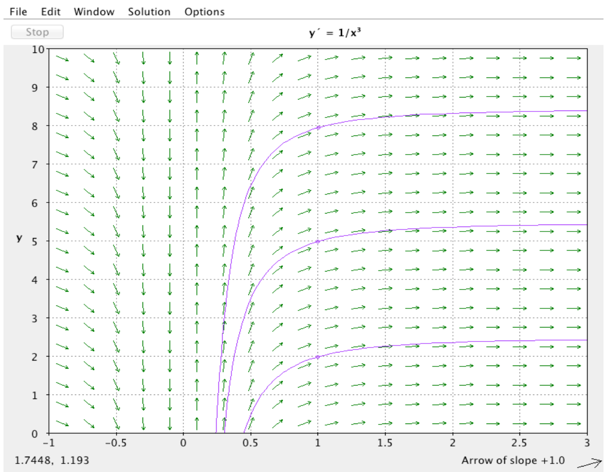
and found the solution $y(x) = \frac{x^2}{2} + 1$. Here's the slope field with this and several other solution graphs drawn in. Note, the slopes are $m(x, y) = x$ in this example. These pictures can be made with the internet application "dfield".



Slope field picture for the IVP in <u>Exercise 1</u> from Friday's notes.

$$f'(x) = \frac{1}{x^3}$$
$$f(1) = 5$$

Is the picture consistent with the solution we find?



<u>Separable Differential Equations</u> So far, all of our differential equations have been ones we can solve via antidifferentiation,

$$\frac{dy}{dx} = f(x)$$
$$\Rightarrow y = \int f(x)\, \mathrm{d}x.$$

For many first order DE's the specified slope of the solution $y(x)$ depends not only on the variable x but also on the value of the function y:

$$\frac{dy}{dx} = m(x, y).$$

If the slope function $m(x, y)$ is a product (or quotient) of a function of x times a function of y, then we can't solve directly by integration, but we can still use the chain rule backwards (and a method called "separation of variables") to solve them. If the DE is

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

then the mathematically rigorous way to find the solution is to rewrite the DE as

$$g(y)y'(x) = f(x).$$

So if $G'(y) = g(y)$, $F'(x) = f(x)$, we can rewrite this as

$$G'(y(x))y'(x) = F'(x)$$

and use the chain rule in reverse to antidifferentiate with respect to x:

$$\mathrm{D}_x G(y(x)) = \mathrm{D}_x F(x)$$
$$\Rightarrow G(y(x)) = F(x) + C.$$

This yields y implicitly defined as a function of x, from which one may or may not be able to explicitly solve for $y(x)$.

Differential magic lets us get to the same correct answer more smoothly

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

$$g(y)dy = f(x)dx$$

$$\int g(y)\ \mathrm{d}y = \int f(x)\ \mathrm{d}x$$

$$G(y) + C_1 = F(x) + C_2$$

$$G(y) = F(x) + C$$

<u>Exercise 1)</u> Solve the differential equation initial value problem

$$\frac{dy}{dx} = x\,y^2$$

$$y(0) = 1$$

using separation of variables. Compare to the slope field picture below.