Math 1210-001
Friday Mar 25
WEB L112

Finish 3.8, antidifferentiation, and begin 3.9 differential equations (which we will finish on Monday).

- First finish Wednesday's notes - We were about to discuss Theorem B on page 4.

- Why the method of "u-substitution" works to do the chain rule backwards in antidifferentiation problems:

The table entry: If $F'(u) = f(u)$ then

$$\int f(g(x))g'(x)\,\mathrm{d}x = F(g(x)) + C.$$

This is true because in this case $\mathrm{D}_x\big(F(g(x)) = F'(g(x))g'(x) = f(g(x))g'(x)$.

You get the same answer using differential "magic" (and often this makes doing the problem easier). Let's check:

$$\int f(g(x))g'(x)\,\mathrm{d}x$$

Substitute

$$u = g(x)$$
$$du = g'(x)dx$$

into the original integral:

$$\int f(u)\,\mathrm{d}u$$
$$= F(u) + C$$
$$= F(g(x)) + C.$$

That's the right answer, so we can use this method to help

Exercise 1) Use $u-$ substitution as an aid in finding

$$\int \frac{36}{(3\,t+2)^3}\,\mathrm{d}t.$$

3.9 Differential Equations

Definition: A differential equation is an equation involving an unknown function, e.g. $y = f(x)$ along with some of its derivatives. Further requirements on the function $f(x)$ may also be specified. Solving the differential equation means finding the solution(s) $y = f(x)$ which make the equation true. Differential equations arise naturally as descriptions of how nature/engineering work (physics, biology, chemistry, engineering etc.). Many of you will take the class Math 2250, which is mostly about this topic.

Example Solve the differential equation with specified initial condition

$$\frac{dy}{dx} = x$$

$$y(0) = 1$$

solution: We want a particular antiderivative of the function "x". All antiderivatives are given by

$$\int x \, dx = \frac{x^2}{2} + C$$

so our solution will be

$$y = \frac{x^2}{2} + C.$$

In order to satisfy the initial condition we must have

$$y(0) = 1 = \frac{0^2}{2} + C = C.$$

So our solution to this problems is

$$y(x) = \frac{x^2}{2} + 1.$$

Exercise 2) Find the function $f(x)$ defined for $x > 0$ for which

$$f'(x) = \frac{1}{x^3}$$

$$f(1) = 5.$$

Exercise 3) Find a function $y = g(x)$ so that

$$g''(x) = 6x + 4$$
$$g(1) = 0$$
$$g'(1) = 0$$

Exercise 4) vertical motion revisited: Newton's law of gravitational acceleration for vertical motion near the surface of the earth is that all objects accelerate equally (discounting friction), namely the height function $y(t)$ satisfies

$$y''(t) = -g$$

where "up" is the positive y-direction, and $g \approx 32\ \frac{ft}{s^2} \approx 9.8\ \frac{m}{s^2}$ is the acceleration of gravity near the surface of the earth. Use this differential equation and antidifferentiate twice, to find the formula for the height function that we were using earlier in the course. Write $y(0) = y_0$, $y'(0) = v_0$.

Antidifferentiation rules to know:

$$\int u^n \, \mathrm{d}u = \frac{u^{n+1}}{n+1} + C \qquad (n \neq -1)$$ power rule in reverse

$$\int \cos(u) \, \mathrm{d}u = \sin(u) + C$$

$$\int \sin(u) \, \mathrm{d}u = -\cos(u) + C$$

$$\int f'(x) \, \mathrm{d}x = f(x) + C$$

$$\mathrm{D}_x\left(\int f(x) \, \mathrm{d}x\right) = f(x)$$

$$\int k f(x) \, \mathrm{d}x = k \int f(x) \, \mathrm{d}x$$ constant multiple rule in reverse

$$\int f(x) + g(x) \, \mathrm{d}x = \int f(x) \, \mathrm{d}x + \int g(x) \, \mathrm{d}x$$ sum rule in reverse

$$\int f'(g(x)) g'(x) \, \mathrm{d}x = f(g(x)) + C$$ chain rule in reverse

$$\int g(x)^n \, g'(x) \mathrm{d}x = \frac{1}{n+1} g(x)^{n+1} + C$$ generalized power rule, special case of previous rule

$$\int f(x) g'(x) \, \mathrm{d}x = f(x) g(x) - \int f'(x) g(x) \, \mathrm{d}x$$ product rule in reverse (called integration by parts -see Math 1220)