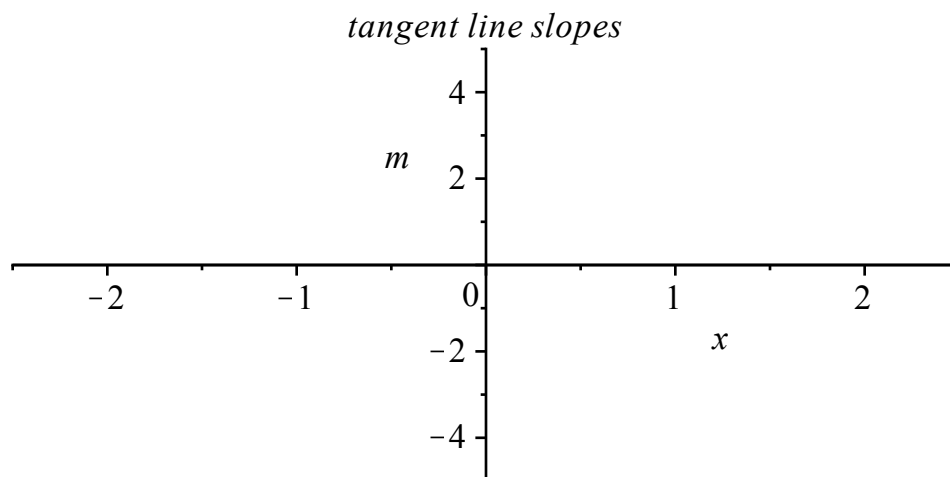
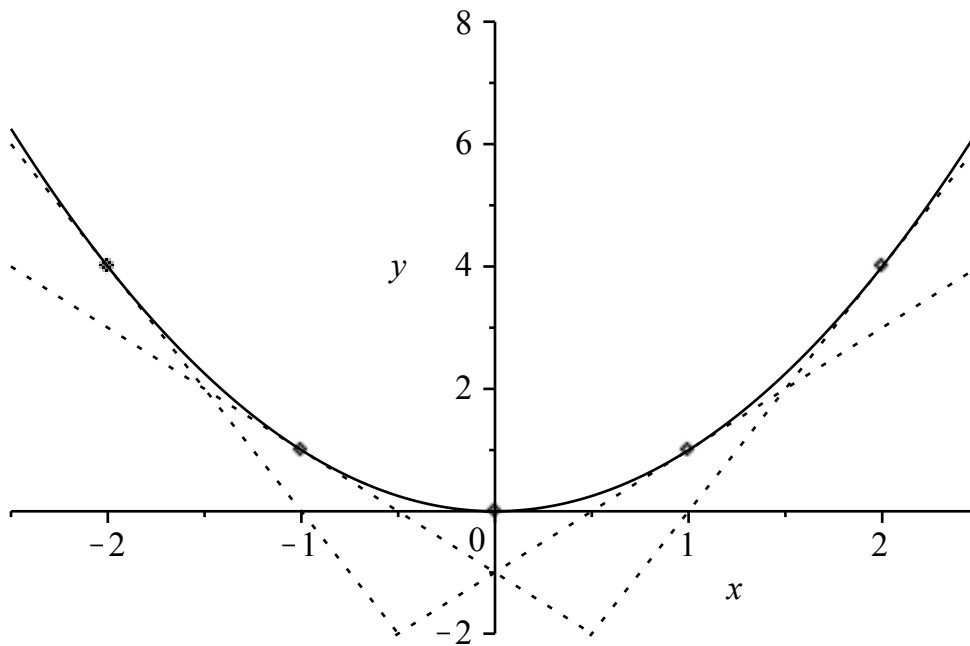


Math 1210-001  
Tuesday Jan 26  
WEB L110

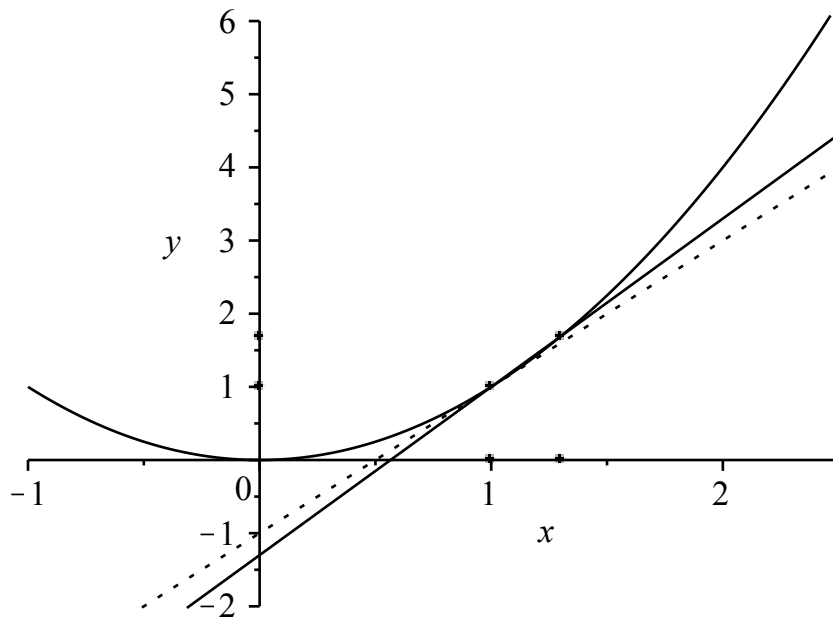
### 2.1-2.2 Introduction to Derivatives.

We've already discussed an example of "slopes of graphs" and "rates of change" - these concepts can be addressed algebraically using limits of "slopes of secant lines" or equivalently "average rates of change". Let's review and expand on that discussion...

Exercise 1a Below is a plot of the graph of  $y = x^2$ , together with 5 tangent lines to the graph. (The points have  $x$ -coordinates  $-2, -1, 0, 1, 2$ .) For each tangent line to the graph, estimate the slope and write the equation of the tangent line. In the second picture plot  $(x, m)$  for each of the 5  $x$ -values, where  $m$  is the corresponding tangent line slope.



1b) The algebraic way to compute the slope of the tangent line via a limit, say at  $x = 1$ , is as a limit of secant line slopes. Set this up, compute the limit, and compare to page 1. This picture will help:



Note, for  $f(x) = x^2$ , the limit we end up computing is

$$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}.$$

1c) We can compute all the tangent line slopes at once, for points on the graph  $y = f(x)$  having horizontal coordinates "x", by computing

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

Note: "x" is a constant in this computation. What do we get, for  $f(x) = x^2$ ? Does our formula for the slope of the graph at different points agree with 1a)? Complete the second picture on page 1.

The mathematical discussion of secant line and tangent line slopes can be rephrased as a discussion about average rates of change and instantaneous rates of change. This becomes more clear if we include units in our discussion.

Exercise 2) From physics, and neglecting friction, the height of an object shot vertically from the surface of the earth, and with an initial velocity of  $64 \frac{ft}{sec}$ , from an initial height of  $0 ft$  above the ground is given by the height function

$$g(t) = -16t^2 + 64t \text{ ft.}$$

a) The average velocity (or, average rate of change of the position function) of the object on an interval  $[c, c + h]$  is defined as the (signed) distance traveled, divided by the time it took, i.e.

$$v_{av} = \frac{g(c+h) - g(c)}{h} \frac{ft}{sec}$$

Compute the average velocity for our example, on the time interval  $1 \leq t \leq 2$  sec. Then compute it for the time interval  $1 \leq t \leq 1.1$ .

b) The instantaneous velocity at time  $c$  is the limit of the average velocities, as the time interval shortens to zero:

$$v(c) := \lim_{h \rightarrow 0} \frac{g(c+h) - g(c)}{h} \frac{ft}{sec}$$

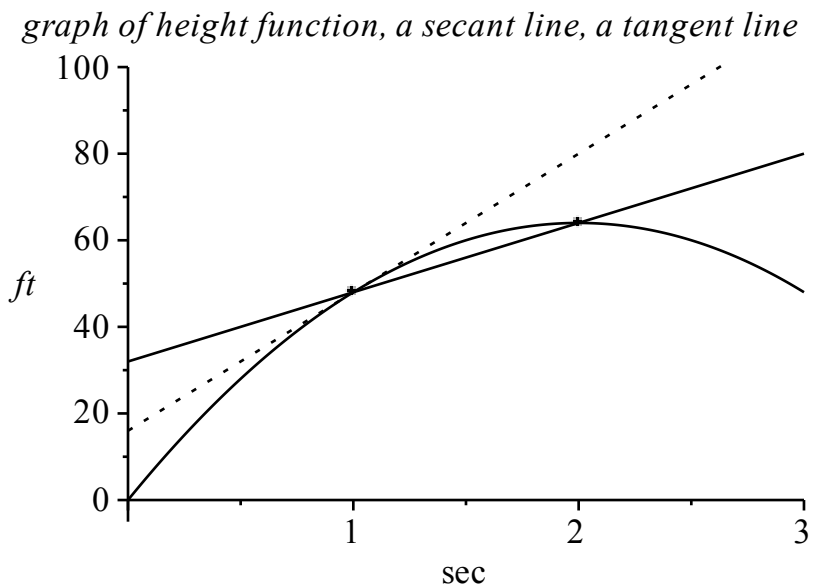
Compute the instantaneous velocity at time  $t = 1$  sec, for position function  $g(t)$  as above. Compare to your approximation in part a.

c) Compute the (instantaneous) velocity for arbitrary  $t$ , i.e.

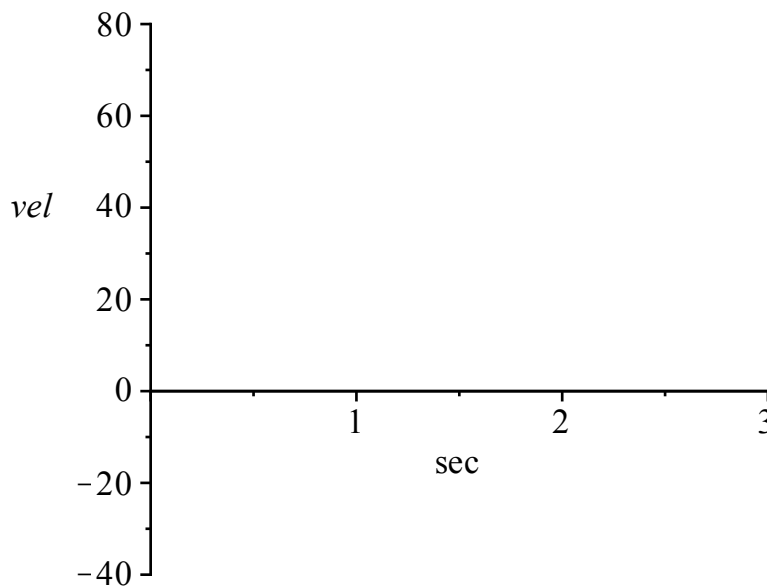
$$v(t) = \lim_{h \rightarrow 0} \frac{g(t+h) - g(t)}{h} \frac{ft}{sec}$$

Compare to a,b.

d) We can interpret the previous page graphically, and we see that the discussion exactly parallels the "secant slope and tangent line slope" discussion. (But, we didn't need the graph picture to complete the algebraic discussion.)



e) Graph the velocity function, and compare to the graph of the height function:



f) The (instantaneous) acceleration at time  $t$  is defined to be the limit of average accelerations:

$$a(t) = \lim_{h \rightarrow 0} \frac{v(t+h) - v(t)}{h}.$$

Compute the acceleration function, for our velocity function. What are its units?

Definitions and summary:

1) For a function  $f(x)$  defined on an open interval containing  $c$ , the derivative of  $f$  at  $c$  is denoted by  $f'(c)$  ("f prime at c") and is defined as

$$f'(c) := \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h},$$

provided the limit exists. If the limit above exists, we say that  $f$  is differentiable at  $c$ . If the limit does not exist, we say that  $f$  is not differentiable there.

2) The difference quotients

$$\frac{f(c+h) - f(c)}{h}$$

represent the slopes of secant lines on the graph of  $f$ . Equivalently, they are the average rates of change of  $f$  on the intervals  $c \leq x \leq c+h$ . The units for the difference quotients are  $\frac{\text{output units}}{\text{input units}}$ , e.g.  $\frac{\text{ft}}{\text{sec}}$ .

These are also the units for  $f'(c)$ .