Math 1210-001
Friday Jan 22
WEB L110

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1.6 Continuous functions: Finish Wednesday's notes about continuous functions, before starting today's. Here's a filled-in version of Theorem D, which we will use to review:

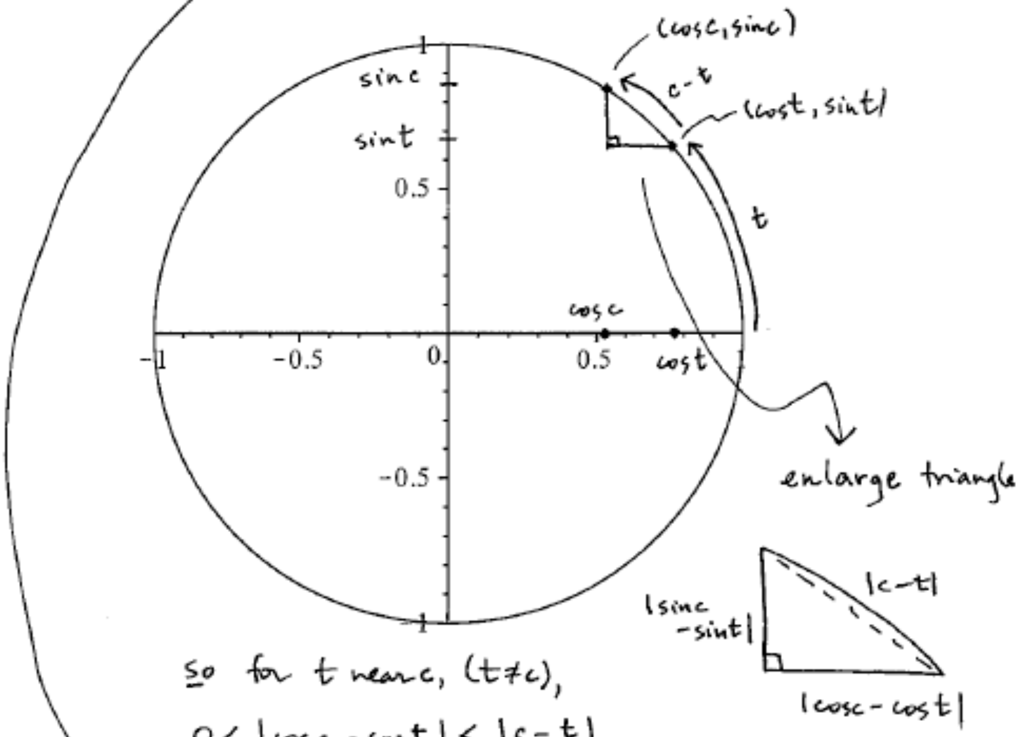
Theorem D (Continuity of trigonometric functions) The $\sin(x)$ and $\cos(x)$ functions are continuous at every real number $c$. The functions $\tan(x)$, $\cot(x)$, $\sec(x)$, $\csc(x)$ are continuous at every real number $c$ in their domains.

Reason: We can deduce the theorem once we show that $\sin(x)$ and $\cos(x)$ are continuous functions, since the other trig functions are obtained from those and the function 1, by taking various quotients...so Theorem C (for quotients) applies.

To show that $\sin(t)$ and $\cos(t)$ are continuous, we must show that

$$\lim_{t \to c} \cos(t) = \cos(c)$$
$$\lim_{t \to c} \sin(t) = \sin(c).$$

We'll use the following diagram to carefully convince ourselves, and recall a bit of trigonometry basics:




so for t near c, $(t \neq c)$,

$$0 < |\cos c - \cos t| < |c-t|$$
$$0 < |\sin c - \sin t| < |c-t|$$

so, t close to c $\Rightarrow$ cos t close to cos c & sin t close to sin c

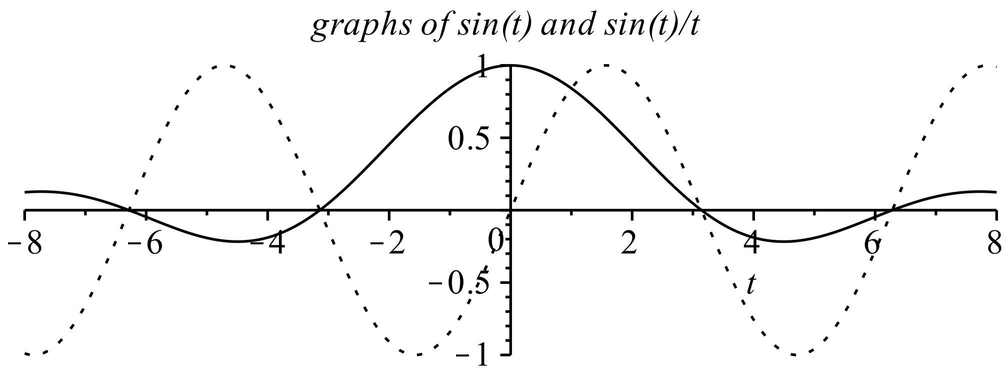
For after we finish Wednesday's notes:

Here's another example of the squeeze theorem. This particular limit will be very important to us in the next chapter.

Exercise 1 Show that

$$\lim_{t \to 0} \frac{\sin(t)}{t} = 1.$$

Before we use the squeeze theorem and geometry to understand this fact, note that the graph of $\frac{\sin(t)}{t}$ (solid graph below) is consistent with it:



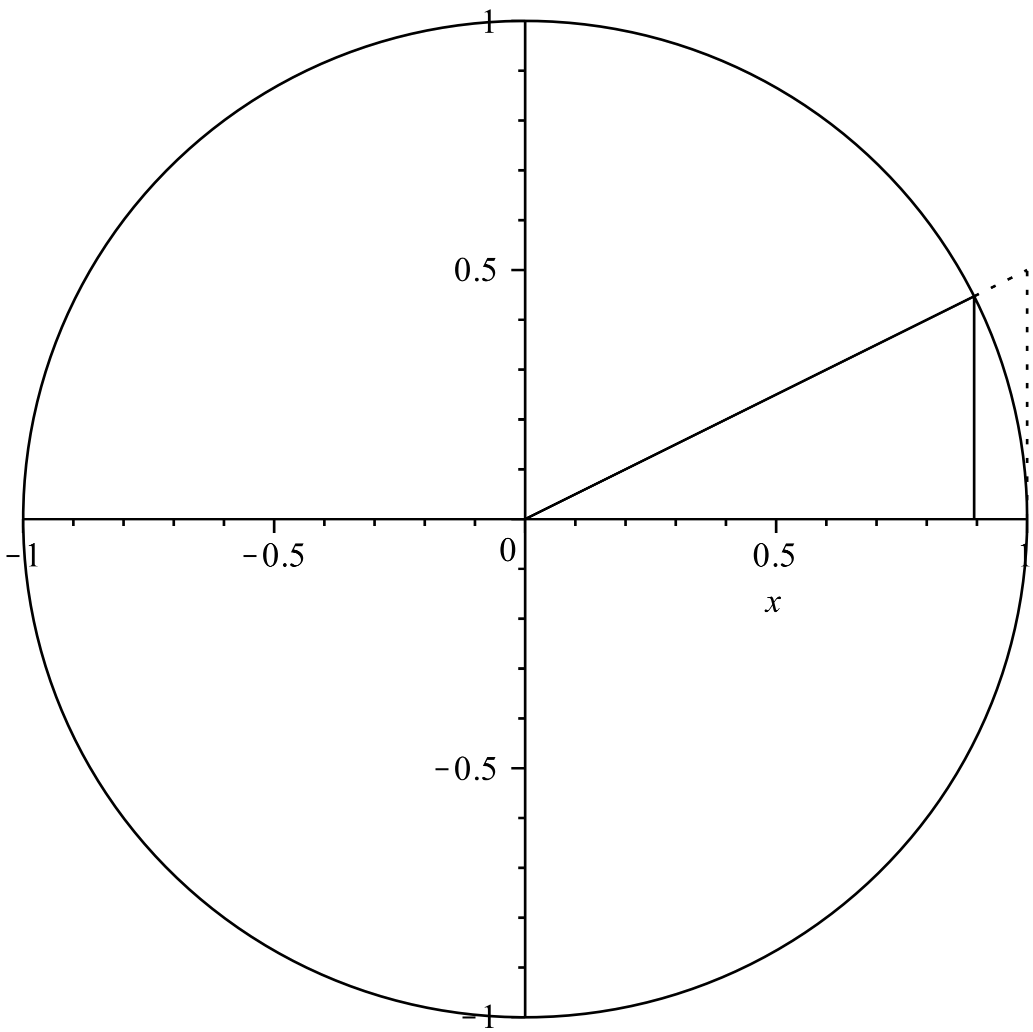

Notice that $f(t) = \frac{\sin(t)}{t}$ is an even function, namely $f(-t) = f(t)$, so to show

$$\lim_{t \to 0} \frac{\sin(t)}{t} = 1$$

It suffices to verify the right-hand limit statement.

$$\lim_{t \to 0+} \frac{\sin(t)}{t} = 1$$

We'll use the unit circle diagram on the next page, and compare the areas of three regions: a smaller right triangle, contained inside a sector of the unit circle, contained in a somewhat larger right triangle that is similar to the first one.


1
0.5
−0.5
−1
−1
−0.5
0
0.5
1
x

Exercise 2 Determine the points of discontinuity of $f(x) = \dfrac{\sin(x)}{x(2-x)^2}$. Classify each point of discontinuity as removable or non-removable.

Summary of the week: Limits with infinity; continuous functions: definition and meaning of continuity at a point, of continuity on intervals, discontinuity; how to identify where functions are continuous and discontinuous using our limit and continuity theorems.

If there's time we'll work some more examples identifying where functions are continuous and discontinuous, before the weekly quiz.