Math 1210-1
Tuesday Jan 11
WEB L110

§1.1 Introduction to Limits

text says "Calculus is the study of limits"
and that's one thing that Calculus is

$\lim_{x \to c} f(x) = L$ "the limit as x approaches c of f(x) equals L"

means, roughly speaking, that when x is "near" c (but $\neq c$), then f(x) is "near" L.

and, we can make f(x) as close to L as we want, if we require x to be close enough to c.

Exercise 1: $\lim_{x \to 1} \frac{x^3 - 1}{x - 1} \stackrel{?}{=} \boxed{}$

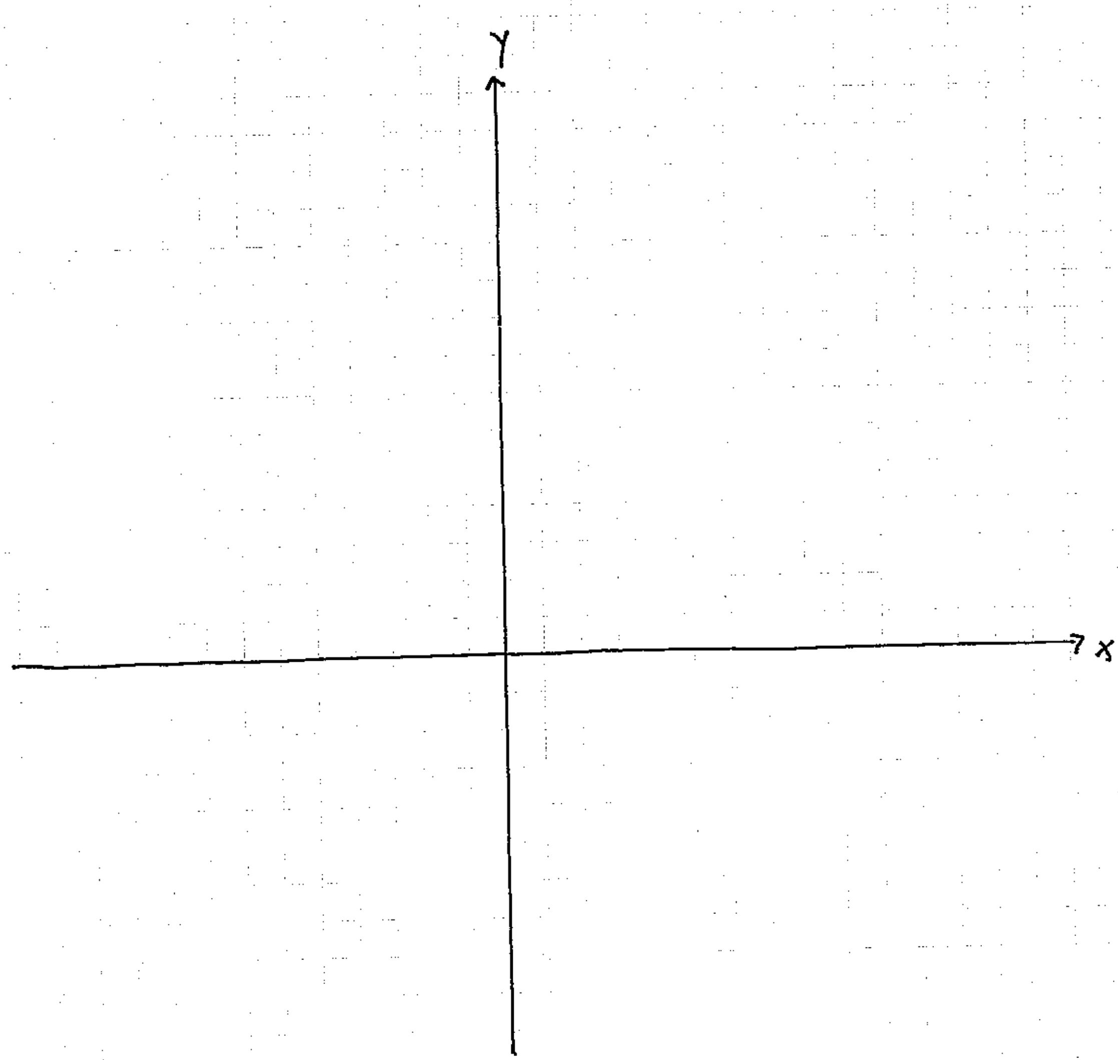
1a) Use your calculator to fill in the table below, and guess the limit

x	f(x)
2	
0	
1.1	
.9	
1.01	
.99	
1	

1b) Do long division on $\frac{x^3-1}{x-1}$, to "verify" your guess in (1a)

1c) Sketch the graph of $\frac{x^3-1}{x-1}$

and explain what the limit computation on page 1 means graphically.



2) The "unit step function" also known as the "Heaviside function" is defined by

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

a) Sketch the graph of $y = u(t)$

b) Does $\lim_{t \to 0} u(t)$ exist?

<u>Def</u> One-sided limits

$\boxed{\lim_{x \to c^+} f(x) = L}$ " the limit as x approaches c from the right (i.e. the positive side of c) equals L"

means, roughly speaking, that when x is close to c, and $x > c$, then $f(x)$ is close to L

<u>and</u> we can make $f(x)$ as close to L as we want, by requiring x to be close enough to c, and $x > c$.

$\boxed{\lim_{x \to c^-} f(x) = L}$ " the limit as x approaches c from the left (i.e from the negative side of c) equals L"

means that when x is close enough to c, with $x < c$, then $f(x)$ is close to

3) For the unit step function $u(t)$ is 2,

discuss

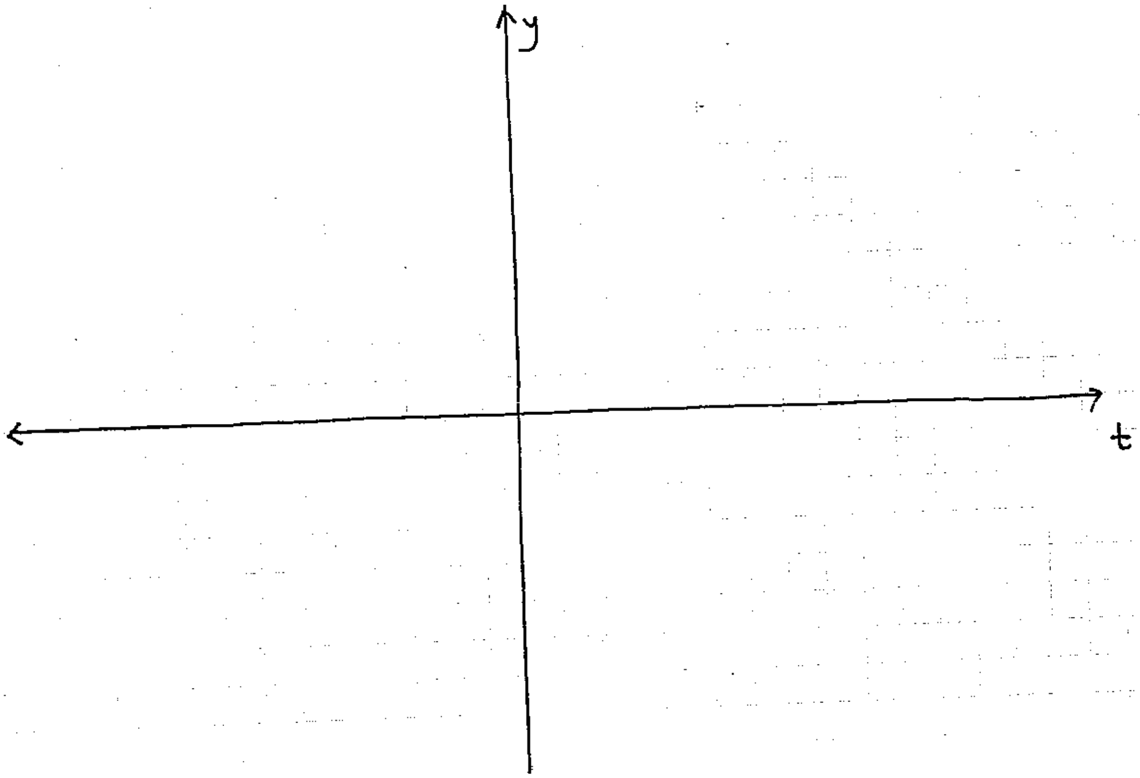
$\lim_{t \to 0^-} u(t)$

$\lim_{t \to 0^+} u(t)$

Exercise 4

a) Sketch the graph of the piecewise-defined function

$$g(t) = \begin{cases} -2, & -\infty < t < -3 \\ 1, & t = -3 \\ -t^2+9, & -3 < t < 3 \\ 2t-6, & t \geq 3 \end{cases}$$



b) Discuss existence of limits and one-sided limits of $g(t)$ at

a) $t = -3$

b) $t = 1$

c) $t = 3$

c) Discuss

Theorem A $\lim_{x \to c} f(x) = L$ if and only if both of $\lim_{x \to c^+} f(x) = L$ and $\lim_{x \to c^-} f(x) = L$