Math 1210-001
Monday Feb 8
WEB L112

2.4: Derivatives of trig functions

so far we know the differentiation shortcuts

$D_x(x^n) = n\,x^{n-1}, n \in \mathbb{Z}.$ (power rule; includes the three special cases above.)
$D_x(f(x) + g(x)) = D_x(f(x)) + D_x(g(x))$ (sum rule)
$D_x(k f(x)) = k\,D_x(f(x))$ if k is a constant. (constant multiple rule)
$D_x(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$ (product rule)
$D_x\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$. (quotient rule)

Today we will see why

$D_x(\sin(x)) = \cos(x)$
$D_x(\cos(x)) = -\sin(x).$

Combining these shortcuts with the quotient rule, we'll get the derivative function shortcuts for $\tan(x), \cot(x), \sec(x), \csc(x)$.

(And, you'll want to memorize all of them so that you can work more quickly on applications problems that we will get to eventually. Tomorrow we'll start to study and use the most important shortcut of all, for derivatives of compositions of functions. It's called the chain rule.)

What we need from trigonometry to understand the derivatives of sin and cos :

1) The three basic trig identities:

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad \text{(Pythagorean identity),}$$

and the two angle addition identities

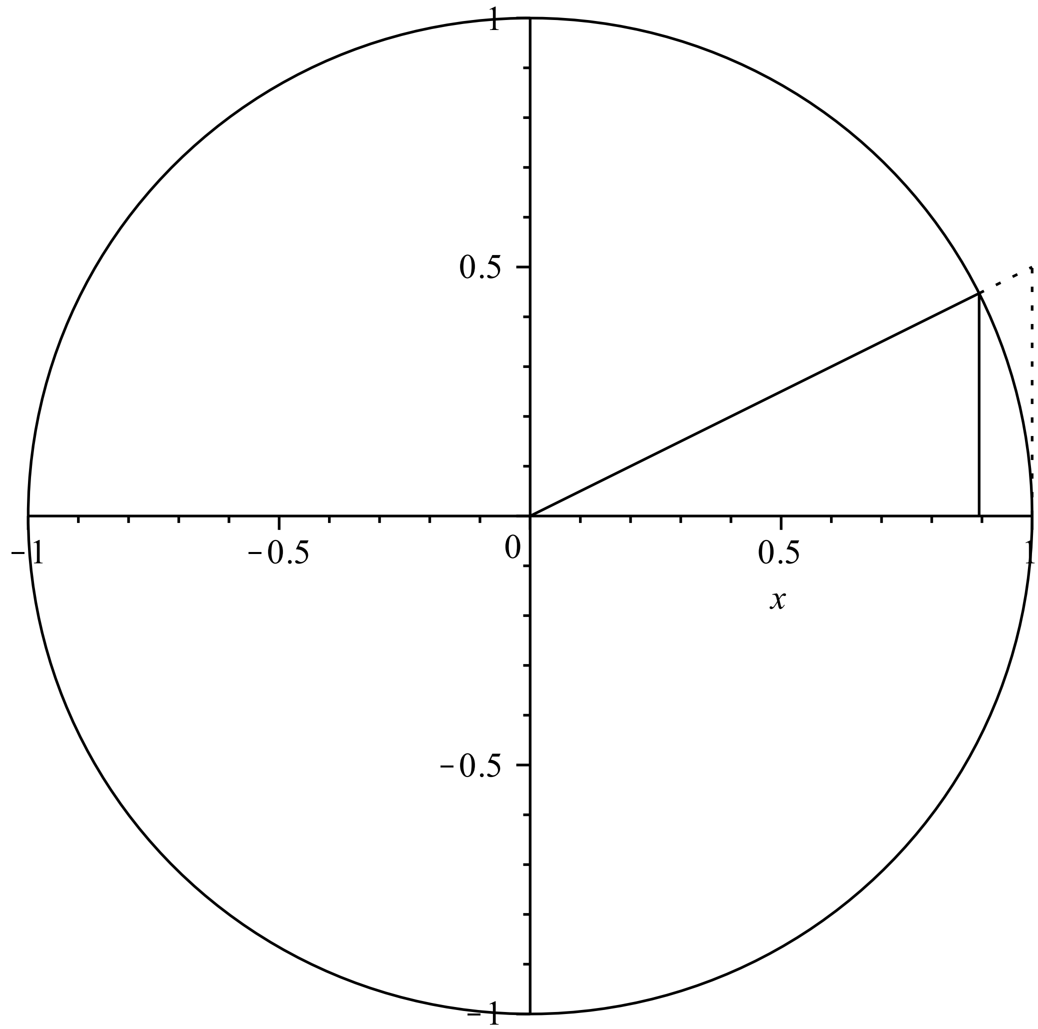
$$\sin(\alpha + \beta) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$$
$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

(I hope to have a lab problem this week which reminds you of where the angle addition identities come from, namely from the geometric ideas behind the formulas for how to rotate points in the plane.)

2) The squeeze theorem limit we proved in the January 22 class note discussion,

$$\lim_{h \to 0} \frac{\sin(h)}{h} = 1.$$

We used this picture, which we will refer to today as well:



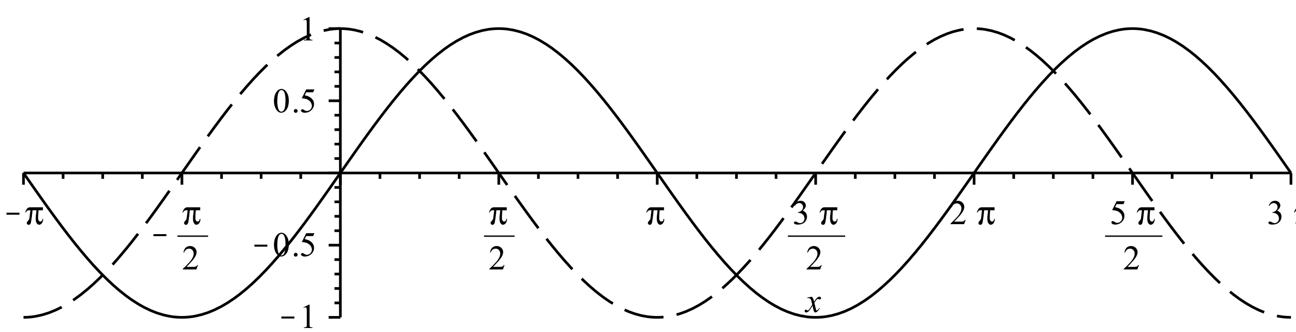
Exercise 1) Use the limit definition of derivative to show

$$D_\theta \sin(\theta) = \cos(\theta)$$
$$D_\theta \cos(\theta) = -\sin(\theta).$$

As a step in the process, you will end up showing these two interesting limits:

$$\lim_{h \to 0} \frac{1 - \cos(h)}{h} = 0$$
$$\lim_{h \to 0} \frac{1 - \cos(h)}{h^2} = \frac{1}{2}.$$

Exercise 2) Discuss the connections between the graphs of sin and cos, having to do with their slope functions.



Exercise 3) Use the quotient rule and Pythagorean identity to verify the derivative rules for the rest of the basic trig functions:

$D_x \tan(x) = \sec^2(x)$
$D_x \cot(x) = -\csc^2(x)$
$D_x \sec(x) = \sec(x)\tan(x)$
$D_x \csc(x) = -\csc(x)\cot(x)$

Exercise 4) Can you figure out the derivative of

$$f(t) = \sin(2\,t)\ ?$$

Does your answer make sense, when you compare the graphs of $\sin(2\,t)$ and $\sin(t)$?
Here's a picture of both of their graphs, which might help us think about this question...and it will tie in to our discussion of the chain rule tomorrow.