Math 1210-001
Wednesday Feb 24
WEB L112

• 3.1-3.2 Finish Tuesday's notes about critical point analysis for finding extreme values of continuous functions on closed bounded intervals. Then continue with today's notes...

Concrete examples of using critical points (for continuous functions on finite closed intervals) to find extreme values:

Exercise 1) Let $f(x) = x^{\frac{2}{3}}$ with specified domain $[-1, 2]$. Use critical point analysis to find the extreme values for f, and where they occur.

Exercise 2) Let $g(x) = x - \sin(2x)$ with specified domain $[0, \pi]$ Use critical point analysis to find the extreme values for f, and where they occur.

More language definitions...

<u>Definition</u> Let f be defined on an interval I (open, closed, or neither). Then

    (i) f is <u>increasing on I</u> means that for all points $x_1 < x_2$ in I, $f(x_1) < f(x_2)$. (In other words, the values of f increase as the input values x increase.)

    (ii) f <u>is decreasing on I</u> means that for all points $x_1 < x_2$ in I, $f(x_1) > f(x_2)$. (In other words, the values of f decrease as the input values x increase.)

    (iii) f is <u>strictly monotone on I</u> means f is either increasing or decreasing on I.

The following theorem makes intuitive sense, based on our understanding that $f'(x)$ is the slope function for the graph of f. We'll discuss a more formal proof later in the chapter:

<u>Theorem A</u> (Monotonicity Theorem) Let f be continuous on the interval I and differentiable at every interior point (i.e. non endpoint) of I. Then
   (i) If $f'(x) > 0$ for all x interior to I then f is increasing on I.
    (ii) If $f'(x) < 0$ for all x interior to I then f is decreasing on I.

<u>Exercise 3)</u> Let $f(x) = x^{\frac{2}{3}}$ with specified domain $[-1, 2]$, from Exercise 1. What are the largest subintervals on which f is monotone? Specify whether f is increasing or decreasing on these subintervals.
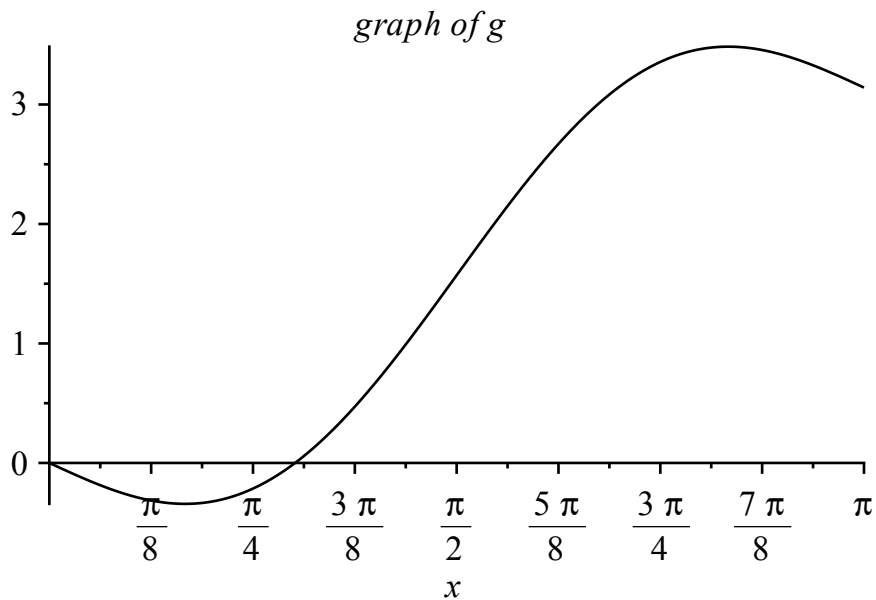

<u>Exercise 4)</u> Let $g(x) = x - \sin(2x)$ with specified domain $[0, \pi]$, from Exercise 2. What are the largest subintervals on which f is monotone?

Here are the graphs of the functions f, g that we've been considering. Compare the graphs to our work so far.

graph of $f(x) = x^{\frac{2}{3}}$ with specified domain $[-1, 2]$:
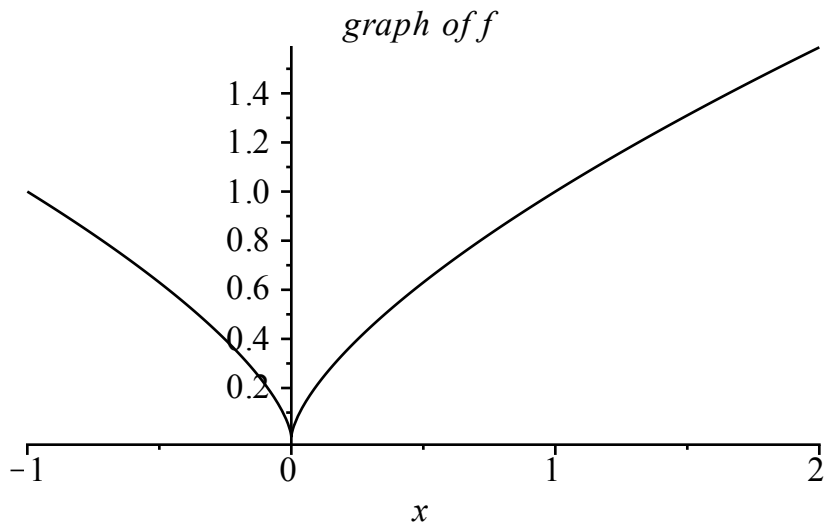
graph of f

graph of $g(x) = x - \sin(2x)$ with specified domain $[0, \pi]$:

graph of g

Definition Let f be differentiable on an open interval I. Then
   (i) f (or the graph of f) is <u>concave up</u> means that the slope function f' is increasing on I.
   (ii) f (or the graph of f) is <u>concave down</u> means that the slope function f' is decreasing on I.

f' increasing: Concave up f' decreasing: Concave down   Concave up   Concave down
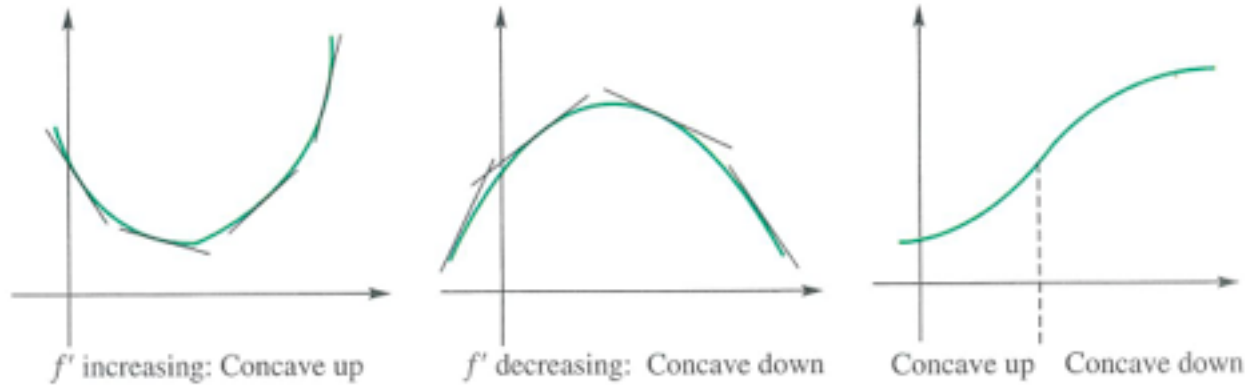
Figure 7

Applying <u>Theorem A</u> from the previous page - but to the slope function f' and its derivative f'' we obtain

<u>Theorem B</u> (Concavity Theorem) Let f be twice differentiable on the open interval I. Then
   (i) If $f''(x) > 0$ for all x in I, then f is concave up on I.
   (ii) If $f''(x) < 0$ for all x in I, then f is concave down on I.

<u>Exercise 5</u> Let $f(x) = x^{\frac{2}{3}}$ with specified domain $[-1, 2]$, from Exercise 1. What are the largest open subintervals on which f concave up and concave down? Compare your algebraic work with the graph on the previous page.

<u>Exercise 6</u> Let $g(x) = x - \sin(2x)$ with specified domain $[0, \pi]$, from Exercise 2. What are the largest open subintervals on which g concave up and concave down? Compare your algebraic work with the graph on the previous page.