

Math 1210-001  
Tuesday Feb 23  
WEB L110

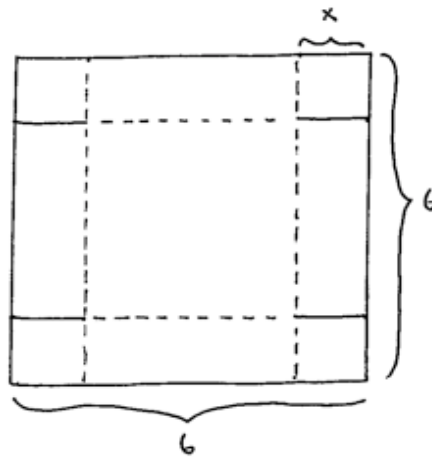
- Work the remaining differentials example from Monday's notes, Exercise 3. Then:

### 3.1 Maxima and minima - also introduction to 3.1-3.5

In science, mathematics, business one seeks (or nature seeks) to optimize an outcome or output - mathematically this means that there are functions which are being maximized or minimized.

Warmup exercise (for the theory described in 3.1)

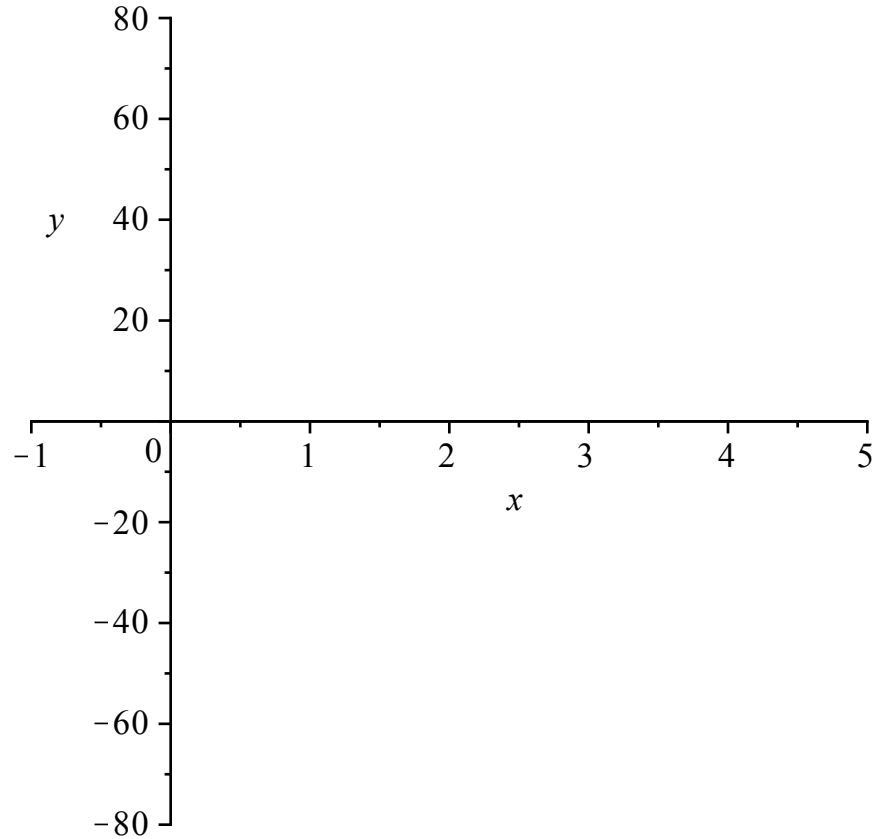
Exercise 1) Your kids want to make paper baskets out of 6 inch by 6 inch squares. They will cut and fold according to the diagram below.

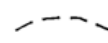
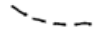


Find  $x$  so that the resulting open-top basket has the most volume possible. Follow these steps:

- 1a) Volume  $V(x) =$
- 1b) Physically reasonable domain for volume function?
- 1c) If the maximum value of  $V$  occurs at some  $x$  inside the domain (i.e. not at an endpoint), then we expect  $V'(x) = 0$  there. Why?
- 1d) Find the dimensions of the open-top box with the largest volume - use logic and 1c.

1e) (This highlights later sections in Chapter 3) Sketch the graph of  $y = V(x)$  for its "natural" domain of  $-\infty < x < \infty$ , i.e only worrying about where the formula makes sense and not about the specific application on the previous page. Include first and second derivative information. Verify that this more complete description of  $y = V(x)$  is consistent with your work in 1d.



$y'$ : information about where graph slope is positive, negative, zero  
 $y''$ : if (slope)' < 0:   
           if (slope)' > 0:   
           "concave up"           "concave down"

Return to section 3.1: We will use some intuitive facts (which are proven in more advanced math classes) to organize our work on "max-min" problems.

First, definitions:

Let  $S$  be the domain of a function  $f$  (typically  $S$  is an interval).

Let  $c$  be a point in  $S$ . Then

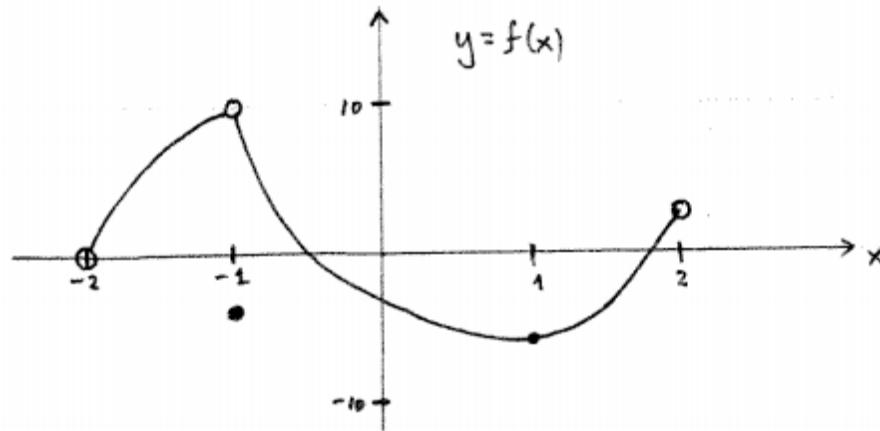
(i)  $f(c)$  is a maximum value of  $f$  on  $S$  means that  $f(c) \geq f(x)$  for all  $x$  in  $S$ .

(ii)  $f(c)$  is a minimum value of  $f$  on  $S$  means that  $f(c) \leq f(x)$  for all  $x$  in  $S$ .

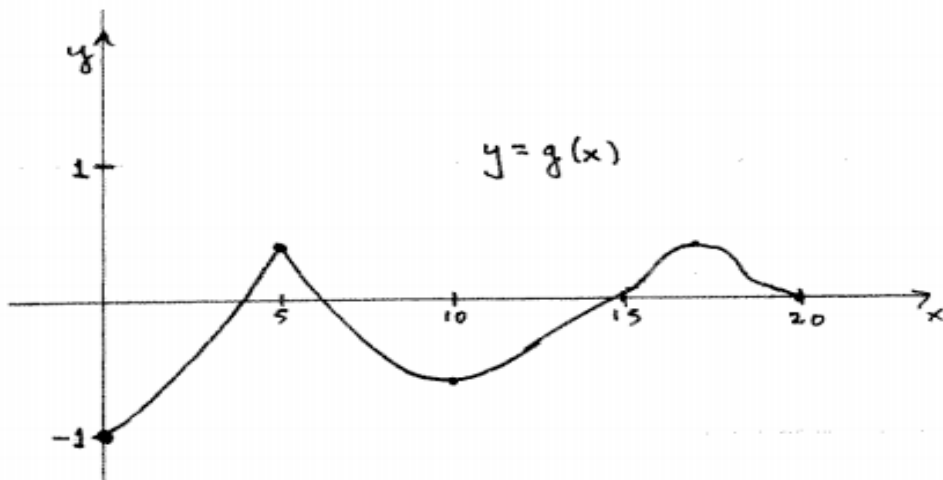
(iii)  $f(c)$  is an extreme value of  $f$  on  $S$  means that  $f(c)$  is either a maximum value or a minimum value of  $f$  on  $S$ .

Exercise 2) From the graphs of the functions shown below and the corresponding domains, find all extreme values, and the input points at which they occur.

2a)  $S = (-2, 2)$ , i.e. the open interval  $-2 < x < 2$ .



2b)  $S = [0, 20]$ , i.e. the closed interval  $0 \leq x \leq 20$ , and  $g(x)$  continuous on  $S$ .



Theorem A) If  $f$  is continuous on a finite closed interval  $[a, b]$  then  $f$  attains a maximum and minimum value (at least once) on its domain (e.g. Exercise 2b).

Theorem B) (limits where extreme values can occur). Let  $f$  be defined on an interval  $I$ . If  $f(c)$  is an extreme value then at least one of the following is true:

- (i)  $c$  is an endpoint of the interval  $I$
- (ii)  $f'(c)$  exists and equals 0 (In this case  $c$  is called a "stationary point" - think physics examples for where that name comes from.)
- (iii)  $f'(c)$  does not exist. (In this case  $c$  is called a "singular point.")

Definition For a function  $f$  defined on an interval  $I$ , points which satisfy (i),(ii), or (iii) are called "critical points" (because knowing those points is critical to finding the extreme values for continuous functions  $f$ ).

Exercise 3) Label all the critical points in Exercises 2a,2b. Compare to Theorems A,B.