Math 1210-001
Monday Feb 22
WEB L112

2.9 Differentials and tangent line approximation

- First - questions about related rates problems and implicit differentiation? WebWork is due this afternoon at 5:00 p.m., and you also have your lab work due Thursday.

- Then begin 2.9

Recall, if $f(x)$ is a function and $y = f(x)$ is its graph, then

$$f'(x) \text{ and } \frac{dy}{dx}$$

are two of our notations for the derivative of f at the point x. If we write

$$f'(x) = \frac{dy}{dx}$$

and think of $\frac{dy}{dx}$ as a quotient, with numerator dy and denominator dx, this motivates the defintion

$$dy := f'(x)\, dx.$$

"dy", "dx" are called <u>differentials</u>.

<u>Exercise 1)</u>

<u>1a</u>) If $y = \sqrt{x^2 + 3x}$ express dy in terms of x and dx.

<u>1b</u>) If $A = \pi r^2$ express dA in terms of r and dr.

<u>1c</u>) If $V = \frac{\pi}{12} h^3$ express dV in terms of h and dh.

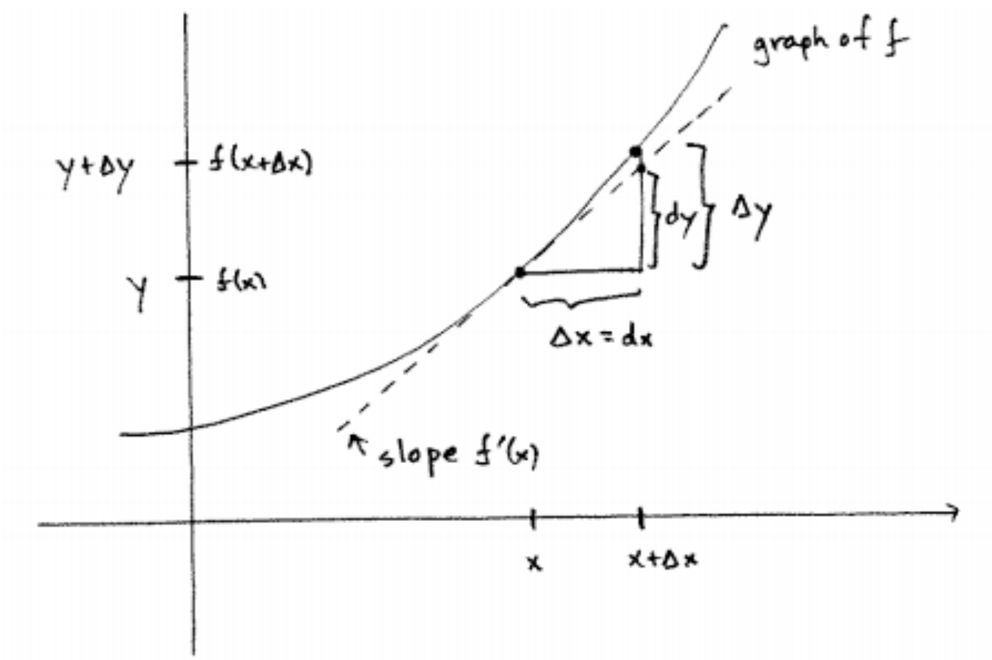
What good are differentials?

Answer: In sections 3.8, 4.4 we will see how differentials guide us in using the chain rule in reverse, for the processeses of antidifferentiation and integration.

In the current section we see how differentials can be used for approximation and error analysis.

Here is a diagram which explains how differentials allow one to approximate function values $f(x+\Delta x)$ in terms of $f(x), f'(x), \Delta x$. This "differential approximation" is also called "tangent line approximation."

In the figure below, we fix "x" and think of "Δx" as varying.



- "x" is fixed. Consider varying small deviations "Δx" for the input variable.
- $\Delta y = f(x+\Delta x) - f(x)$ is the corresponding change in the output variable, i.e.
$$f(x+\Delta x) = f(x) + \Delta y.$$
- Set $dx = \Delta x$, $dy = f'(x)dx$.

Then dy is an approximation to the exact change Δy, that we get by using the tangent line at $(x, f(x))$ to approximate the actual graph of f (for small input deviations).
$$f(x+dx) \approx f(x) + dy.$$

Exercise 2) Consider $y = \sqrt{x}$. Use differentials and approximate

2a) $\sqrt{4.2}$ $(x = 4,\ dx = 0.2)$

(the decimal value is $\sqrt{4.2} = 2.04939\ldots$)

2b) $\sqrt{8.7}$

(the decimal value is $\sqrt{8.7} = 2.949576\ldots$)

Exercise 3) Use differentials to estimate the volume of paint necessary to paint a hemisphere of radius 5 feet, with a layer of paint 0.05 inches thick. Express your answer in gallons. (Hint: $1\,ft^3 = 12^3 in^3$; $7.481\ gal = 1\,ft^3$.) Compare your estimate to the exact value.

Exercise 4) The side lengths of a cube are measured to be $10\ cm \pm 0.1\ cm$. Use differentials to estimate how close the volume is to $10^3\ cm^3$. Compare to exact error estimate.