Math 1210-001
Wednesday Feb 17
WEB L112

2.8 Related rates

In section 2.7 we used identity equations and the chain rule to find the derivatives of functions at given points on their graphs, without needing to know the explicit formulas for the functions. This method is called implicit differentiation. In section 2.8 we use identities involving several different functions (of time) at once, along with the chain rule, in order to figure out how fast one of those functions is changing with respect to time, in terms of how fast the others are. This topic is called related rates.

Exercise 1 (page 135 of text). A balloon is released 150 horizontal feet away from an observer, and rises at a rate of 8 $\frac{ft}{\text{sec}}$. When the balloon is 50 ft high, how fast is the distance from the observer changing? (The general steps to follow in a related rates problem are listed at the bottom of the page. We will follow these steps.)

steps:
1) Construct a diagram with the relevant functions (and constants) labeled.
2) What is known? What is asked for? Express quantities mathematically as function values and/or derivatives.
3) Relate the known and unknown functions with one or more identity equations.
4) Differentiate the identity equation in 3, with respect to time. Most likely this will require use of the chain rule, like in implicit differentiation problems.
5) Using the identity equations and their differentiated versions, use the known function and derivative values to find the unknown one.

Exercise 2) Farmer Jones is growing a pumpkin for a contest. At day 58 of the pumpkin's growth its diameter is 0.8 m and is increasing at a rate of 4.2 cm per day. How fast is its volume changing at that time?

Exercise 3) (#17, page 141) Chris, who is 6 feet tall, is walking away from a street light pole 30 feet high, at a rate of 2 $\frac{ft}{\text{sec}}$.

3a) How fast is his shadow increasing in length when Chris is 24 feet from the pole? 30 feet?

3b) How fast is the tip of his shadow moving? Does this speed depend on time?

3c) To follow the tip of the shadow, at what angular rate must Chris be lifting his eyes when his shadow is 6 feet long? (!!)

Exercise 4) (page 140) Webster City monitors the height of water in its (vertical) cylindrical water tank with an automatic recording device. Water is constantly pumped into the tank at a rate of 2400 $\frac{ft^3}{hour}$. During a certain 12-hour period (beginning at midnight), the water level rose and fell according to the graph in Figure 8 below. If the radius of the tank is 20 feet, at what rate was water being used at 7:00 a.m. ?

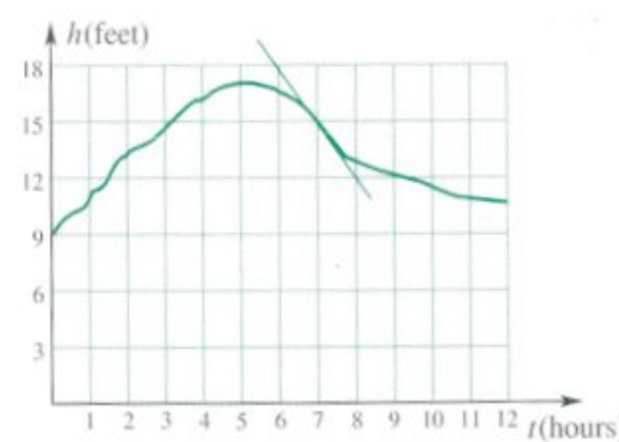

Figure 8