Math 1210-001
Friday Feb 12
WEB L112

2.6 Higher order derivatives, derivatives notation. You may also have questions about current WebWork and lab assignments, so today's notes are relatively short.

Second order derivatives:

If we take the derivative of the derivative $f'(x)$, of a function $f(x)$, we call that the second derivative of *f*, and write it as $f''(x)$.

Example 1 If $f(t)$ is recording the position in meters of an object on a number line, at time *t* seconds, then

$$f'(t)$$

is the rate of change of position with respect to time at time *t*, has units $\frac{m}{s}$ and is also called the velocity $v(t)$. The derivative of the velocity measures the rate of change of velocity with respect to time, has units $\frac{m}{s^2}$, and is also called the acceleration $a(t)$.

$$f'(t) = v(t)$$
$$f''(t) = v'(t) = a(t).$$

Higher order derivatives

We can consider third, fourth, and higher order derivatives as well. Here's a table that lists the different types of notation that occur when we talk about derivatives. The Leibniz notation will be useful coming up - we've introduced it once or twice already, but not really made essential use of it so far.

The table below shows the possible notations, for a function $f(x)$, and its associated graph $y = f(x)$.

Derivative	prime notation	D notation	Leibniz notation
first	$f'(x)$ or y'	$D_x f(x)$ or $D_x y$	$\frac{dy}{dx}$ or $\frac{d}{dx}y$
second	$f''(x)$ or y''	$D_x^2 f(x)$ or $D_x^2 y$ (short for $D_x(D_x f(x))$)	$\frac{d^2y}{dx^2}$ or $\frac{d^2}{dx^2}y$
third	$f'''(x)$ or y'''	$D_x^3 f(x)$ or $D_x^3 y$	$\frac{d^3y}{dx^3}$ or $\frac{d^3}{dx^3}y$
n^{th} derivative	$f^{(n)}(x)$ or $y^{(n)}$ (note the parantheses)	$D_x^n f(x)$ or $D_x^n y$	$\frac{d^ny}{dx^n}$ or $\frac{d^n}{dx^n}y$

Exercise 1) If $y = \sin(2\,t)$, compute

1a) y'.

1b) $D_t^2 y$

1c) $\frac{d^3 y}{dt^3}$

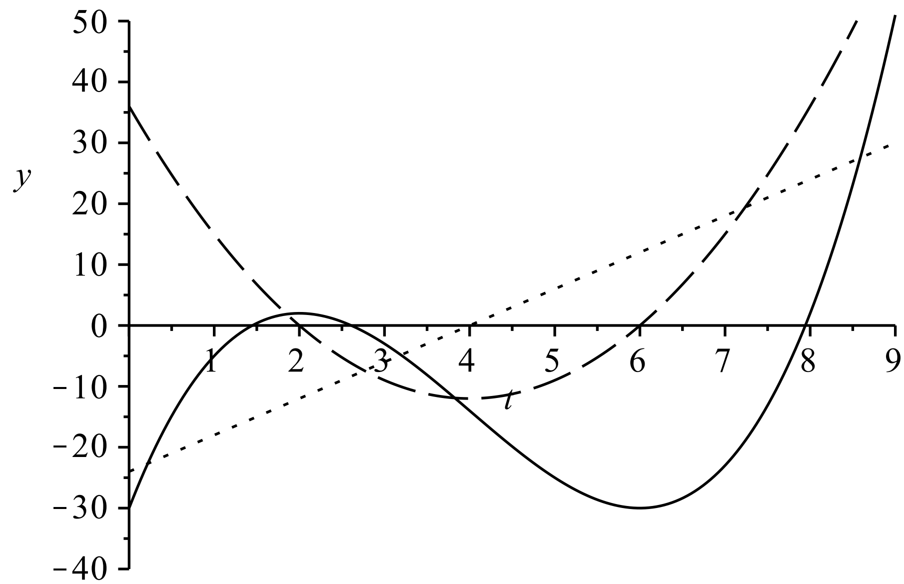
1d) $y^{10}(t)$

1e) $y^{(10)}(t)$.

Are there any WebWork problems on this topic that you'd like to discuss? The last two problems, 19,20 have to do with higher order derivatives.

In terms of applications, most important information about a function is obtained from its formula and from its first and second derivatives. For example, we'll use that sort of information to understand the shape of function graphs in Chapter 3. Here's a preview:

Exercise 2) Here is a graph of a function $f(t) = t^3 - 12 \cdot t^2 + 36 \cdot t - 30$, and its first and second derivative functions. Identify which graph is which. Discuss what the sign of $f'(t)$ has to do with properties of the graph of f. Also what the sign of $f''(t)$ has to do with the shape of the graph of f', and the shape of the graph of f.



Exercise 3) Suppose the $f(t)$ in the first problem was describing an objects location on a number line (marked off in meters), at time t seconds. Describe the object's motion for $0 \leq t \leq 10$. (You had a problem like this on WebWork last week, that you studied with just the first derivative.)

Things to know page.

Differentiation rules:

$D_x(x^n) = n\,x^{n-1},\ n \in \mathbb{Z}.$ (power rule; includes the three special cases above.)

$D_x(f(x) + g(x)) = D_x(f(x)) + D_x(g(x))$ (sum rule)

$D_x(kf(x)) = k\,D_x(f(x))$ if k is a constant. (constant multiple rule)

$D_x(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$ (product rule)

$D_x\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$. (quotient rule)

$D_x(f(g(x)) = f'(g(x))\cdot g'(x)$ (chain rule)

$D_x(\sin(x)) = \cos(x)$

$D_x(\cos(x)) = -\sin(x)$

$D_x \tan(x) = \sec^2(x)$

$D_x \cot(x) = -\csc^2(x)$

$D_x \sec(x) = \sec(x)\tan(x)$

$D_x \csc(x) = -\csc(x)\cot(x)$

Trigonometry identities:

$\cos^2\theta + \sin^2\theta = 1$

$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\sin(\alpha + B) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$.