Math 1210-001
Wednesday Feb 10
WEB L112

2.4-2.5: Chain rule and trig function derivatives drill day. We'll go through the first several exercises together, before letting you test yourselves and practice together on the later problems. The goal is to develop your ability to do these computations quickly and accurately. You may also have some WebWork questions.

$D_x(f(g(x)) = f'(g(x)) \cdot g'(x)$ (chain rule)

(The rate of change of a composition is the product of the rates of change of each function being composed, at the appropriate input values.)
(The derivative of the composition is the derivative of the outer function, evaluated at the inner function value, times the derivative of the inner function.)

<u>Exercise 1)</u> $D_x\,(3x^2+7)^{100}$.

$g(x) = 3x^2+7 \qquad g'(x) = 6x$
$f(u) = u^{100} \qquad f'(u) = 100u^{99}$

$D_x\,(3x^2+7)^{100} = f'(g(x))\,g'(x)$
$= 100\,(3x^2+7)^{99} \cdot 6x \; = \; 600\,x\,(3x^2+7)^{99}$

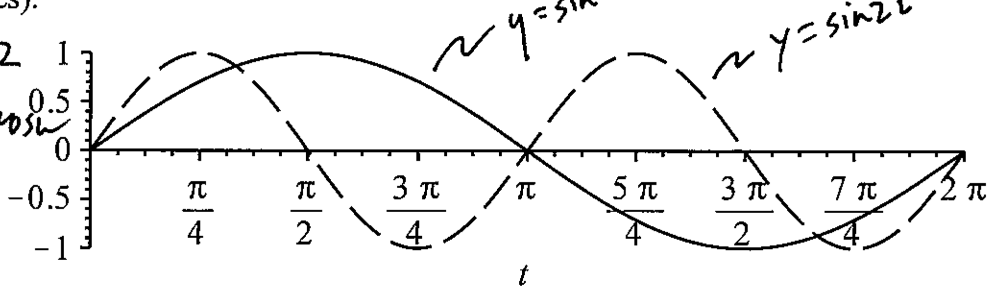
<u>Exercise 2)</u> $D_t(3\cos^2 t + 7)$.

$g(t) = \cos t \qquad g'(t) = -\sin t$
$f(u) = 3u^2+7 \qquad f'(u) = 6u$

$f'(g(t))g'(t) = 6\cos t\,(-\sin t) = -6\cos t \sin t$

<u>Exercise 3)</u> $F'(t)$ for $F(t) = \sin(2t)$ (understand your answer geometrically by looking at the graph below, from Monday's notes).

$g(t) = 2t \qquad g'(t) = 2$
$f(u) = \sin u \qquad f'(u) = \cos u$



$F'(t)$
$= f'(g(t))g'(t) = \cos(2t) \cdot 2$

the factor of 2 corresponds to the fact that the graph of $y = \sin 2t$ is compressed horizontally by a factor of 2, compared to graph of $y = \sin t$, so slopes are twice as steep.

Exercise 4) $G'(1)$ if $G(t) = \underbrace{(t^2+9)^3}_{f}\underbrace{(t^2-2)^4}_{g}$.

$$G' = f'g + fg'$$
$$= 3(t^2+9)^2 \cdot 2t \cdot (t^2-2)^4 + (t^2+9)^3 4(t^2-2)^3 \cdot 2t$$
$$G'(1) = 3 \cdot 100 \cdot 2 \cdot 1 + 1000 \cdot 4(-1)(2) = 600 - 8000 = -7400$$

details: $f(t) = (t^2+9)^3$ similar for $g(t)$.

is a composition $h(k(t))$

$h(t) = t^2+9 \quad h'(t) = 2t$

$k(u) = u^3 \quad k'(u) = 3u^2$

so $f'(t) = h'(k(t)) \cdot k'(t)$

$= 3(t^2+9)^2 \cdot 2t$

Exercise 5) find $f'(3)$ if $f(x) = \left(\frac{x^2+1}{x+2}\right)^3$.

$= h(g(x))$

$g(x) = \frac{x^2+1}{x+2} \qquad g'(x) = \frac{2x(x+2) - (x^2+1)\cdot 1}{(x+2)^2}$

$h(u) = u^3 \qquad\qquad = \frac{x^2+4x-1}{(x+2)^2}$

$h'(u) = 3u^2$

$h'(g(x)) \cdot g'(x)$

$$= 3\left(\frac{x^2+1}{x+2}\right)^2 \cdot \frac{x^2+4x-1}{(x+2)^2}$$

could simplify ...

Exercise 6) What's the symbolic formula for the derivative of a triple composition?

$$D_x f \circ g \circ h(x) =$$

$$D_x f(g(h(x))) = f'(g(h(x))) \cdot D_x g(h(x))$$ 1st chain rule application

$$= f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$ 2nd chain rule application

Exercise 7) $D_t \sin(\cos(t^2 + 5))$.

$$= \cos(\cos(t^2+5)) \cdot (-\sin(t^2+5)) \cdot 2t$$

Exercise 8) $D_t \dfrac{1}{\sin(t)}$ (leftovers... :-))

$$= \frac{0 \cdot \sin t - 1 \cdot \cos t}{(\sin t)^2} = \frac{-\cos t}{\sin t \cdot \sin t} = -\frac{\cos t}{\sin t} \; \frac{1}{\sin t}$$

so $D_t \csc t = -\csc t \cot t$

Exercise 9) $\dfrac{d}{dt}\left(\dfrac{\overbrace{\tan(3t)+1}^{f}}{\underbrace{\cos^3(2t)}_{g}}\right) = \dfrac{f'g - fg'}{g^2}$

$$= \frac{\sec^2(3t) \cdot 3 \cdot \cos^3 2t - (\tan 3t + 1) \cdot 3\cos^2 2t\,(-\sin 2t) \cdot 2}{\cos^6(2t)}$$

Things to know page.

Differentiation rules:

$D_x(x^n) = n x^{n-1}, n \in \mathbb{Z}.$ (power rule; includes the three special cases above.)

$D_x(f(x) + g(x)) = D_x(f(x)) + D_x(g(x))$ (sum rule)

$D_x(kf(x)) = k D_x(f(x))$ if k is a constant. (constant multiple rule)

$D_x(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$ (product rule)

$D_x\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.$ (quotient rule)

$D_x(f(g(x)) = f'(g(x)) \cdot g'(x)$ (chain rule)

$D_x(\sin(x)) = \cos(x)$

$D_x(\cos(x)) = -\sin(x)$

$D_x \tan(x) = \sec^2(x)$

$D_x \cot(x) = -\csc^2(x)$

$D_x \sec(x) = \sec(x)\tan(x)$

$D_x \csc(x) = -\csc(x)\cot(x)$

Trigonometry identities:

$\cos^2\theta + \sin^2\theta = 1$

$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\sin(\alpha + B) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$.