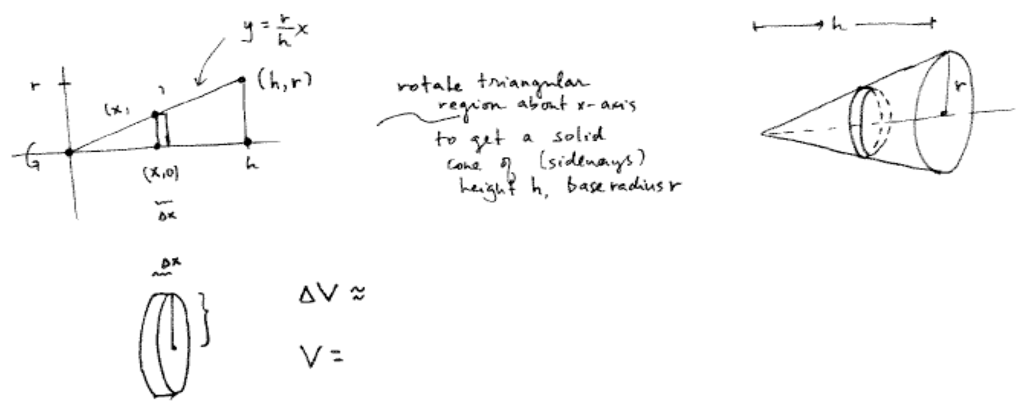
Math 1210-001
Monday Apr 18
WEB L112

- 5.2: Continue volumes by planar-slab slicing. I've imported two examples from last week that we didn't have time for, and we'll do some more as well.
- 5.3: Volumes by cylindrical shells - we'll start this section today if we have time.

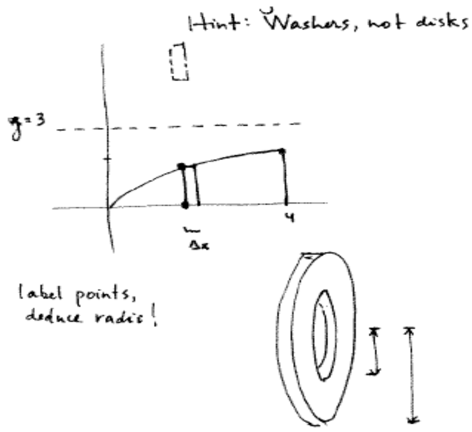
Exercise 1) Derive the formula for the volume of a cone, $V = \frac{1}{3}\pi\, r^2 h$. (Where did that factor of $\frac{1}{3}$ come from?)

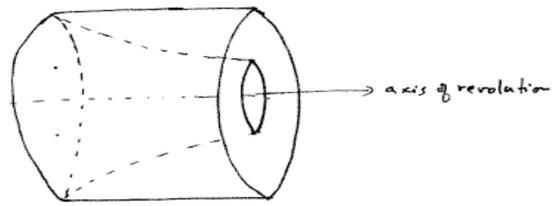


> $\int_0^h \pi \cdot \left(\left(\frac{r}{h} \right) \cdot x \right)^2 \, dx;$

$$\frac{1}{3}\pi r^2 h \qquad \textbf{(1)}$$

Exercise 2) Consider the region in the first quadrant bounded above by the $y = \sqrt{x}$, below by the $x - axis$, and on the right by $x = 4$. Rotate this region about the horizontal line $y = 3$. What is the resulting solid's volume? Set up the integral.





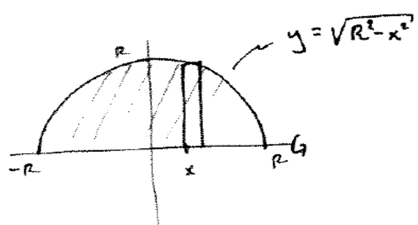
ΔV=

V=

> $\pi \cdot \int_0^4 (9 - x)\, dx;$

$$28\,\pi \tag{2}$$

Exercise 3 (Lab 5g, special case of WebWork #9) Recover the formula for the volume of a solid ball of radius R by considering the solid generated by rotating the upper half disk, bounded above by $y = \sqrt{R^2 - x^2}$ and below by the $x - axis$.

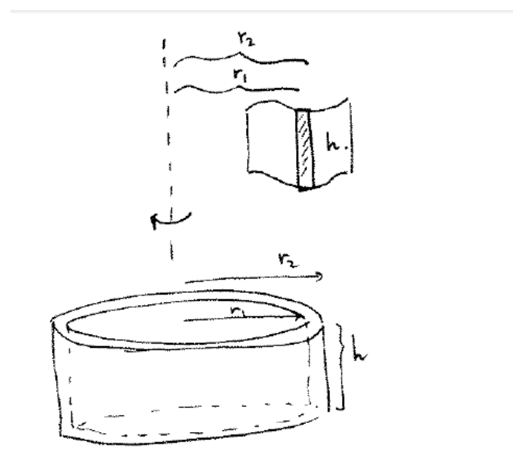


ΔV ≈

V =

• <u>5.3 Volumes by cylindrical shells</u>

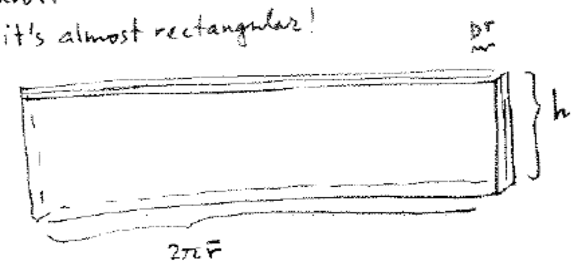
Sometimes for a volume of revolution the, slab methods of section 5.2 are not the most effective and may not even work. There's another way to decompose these volumes - "cylindrical shells." With slabs we chop the region being rotated <u>perpendicular</u> to the axis of revolution, and the chopped pieces create slabs (usually disks or washers) when rotated. If instead we chop the region <u>parallel</u> to the axis of revolution, the rotated regions are cylindrical shells:



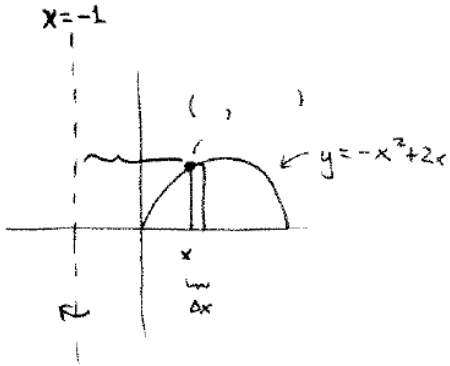
$$\Delta V = \pi(r_2^2 - r_1^2)h$$
$$= \pi\underbrace{(r_2 - r_1)}_{\Delta r}\underbrace{(r_2 + r_1)}_{2\bar{r}}h$$
where $\bar{r} = \frac{r_1 + r_2}{2}$

$$\Delta V = 2\pi\bar{r}h\,\Delta r$$

Easy to remember: cut the cylinder & unroll:
it's almost rectangular!



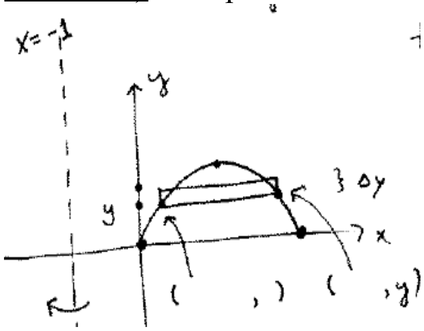
Exercise 4) Consider the region in the first quadrant bounded above by the parabola $y = -x^2 + 2x = -(x-1)^2 + 1$, and below by the $x - axis$. Rotate this region about the line $y = -1$ and compute the volume with shells.



$\Delta V \approx$

$V =$

Exercise 5) Set up the same volume computation with washers instead.



$\Delta V =$

$V =$

Exercise 4 solution:

> $\int_0^2 2\cdot \pi\cdot (x+1)\cdot(-x^2+2\cdot x)\,dx;$

$$\frac{16}{3}\pi \tag{3}$$

>

Exercise 5 solution:

> $\int_0^1 \pi\cdot\left(\left(2+\sqrt{1-y}\right)^2-\left(2-\sqrt{1-y}\right)^2\right)dy;$

$$\frac{16}{3}\pi \tag{4}$$

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