

Math 1210-001  
 Wednesday Apr 13  
 WEB L112

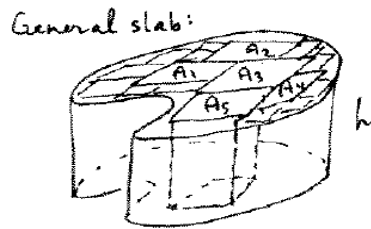
- We'll do Exercise 4 on Monday's notes - work on 4a with your classmates before and during the first few minutes of class. I'll put some of the details of what to do on the blackboard. Then we'll discuss all facets of problem 4 - including how to compute areas of regions where the boundary curves are given by  $x$  is a function of  $y$  rather than the reverse. This will likely take at least the first third of the lecture, and complete our discussion of section 5.1

- 5.2: Volumes by planar-slab slicing.

Step 1: The volume of any "slab" (cylinder with arbitrary cross section) with cross section area  $A$ , and with height  $h$  is

$$V = A h \text{ (cubic units)}$$

This picture explains why:

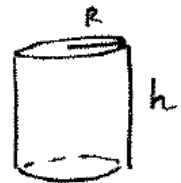


$$A = \sum_{i=1}^{\infty} A_i$$

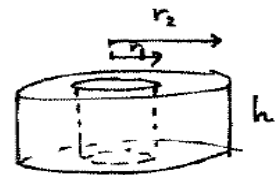
$$V = \sum_{i=1}^{\infty} A_i h = h \sum_{i=1}^{\infty} A_i = h A$$

Exercise 1) As examples of step 1, what are the following volumes?

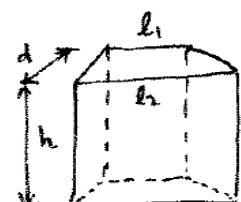
1a) Solid cylinder, cross section is a disk of radius  $R$ , height  $h$ .  $V =$



1b) Washer (hollow core), with inner radius  $r_1$ , outer radius  $r_2$ , height  $h$ .  $V =$

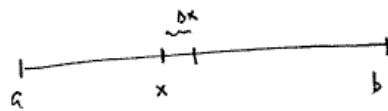
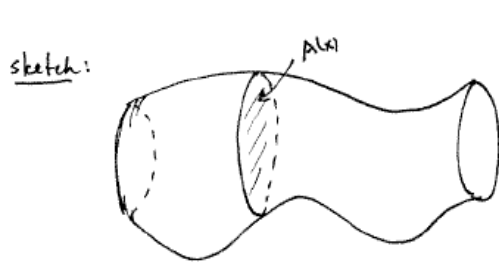


1c) Trapezoidal prism, with cross section that is a trapezoid of "height"  $d$ , parallel side lengths of  $l_1, l_2$ ; and height  $h$ .  $V =$



Step 2: Add Calculus!

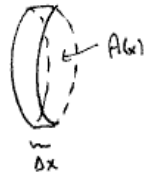
Consider a 3-dimensional solid located between  $a \leq x \leq b$ , with continuously varying cross-sectional area  $A(x)$ . We wish to compute its volume. Slice the solid into slabs, use Riemann sums to write down the approximate volumes, and take the limit as the slab thicknesses approach 0:



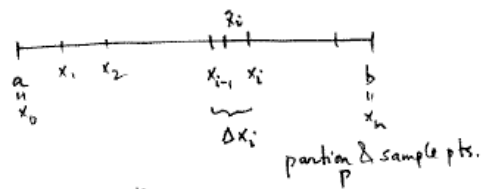
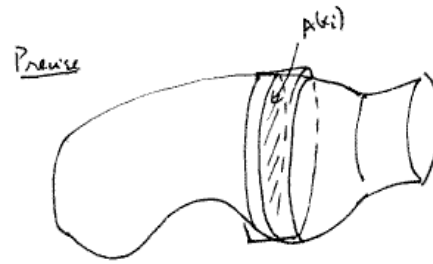
All we need to write down to remind us of the details on the right:

$$\Delta V \approx A(x) \Delta x$$

the area between  $x$  &  $x+\Delta x$



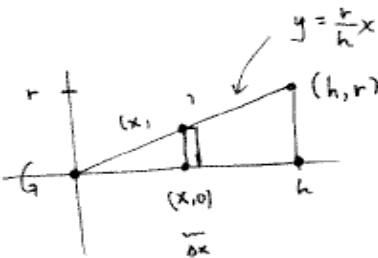
$$V = \int_a^b A(x) dx$$



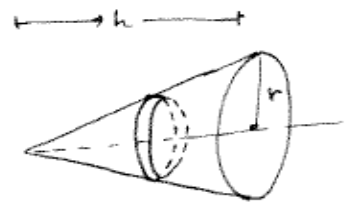
$$V \approx \sum_{i=1}^n A(x_i) \Delta x_i = R_P$$

volume  $V$  defined to be  $\lim_{\|P\| \rightarrow 0} R_P = \int_a^b A(x) dx$   
by definition of definite integral.

Exercise 2) Derive the formula for the volume of a cone,  $V = \frac{1}{3} \pi r^2 h$ . (Where did that factor of  $\frac{1}{3}$  come from?)



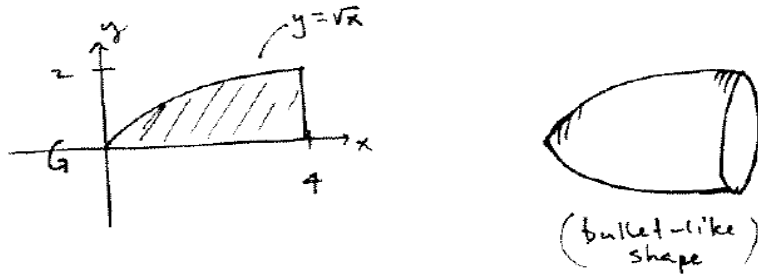
rotate triangular region about x-axis to get a solid cone of (sideways) height  $h$ , base radius  $r$



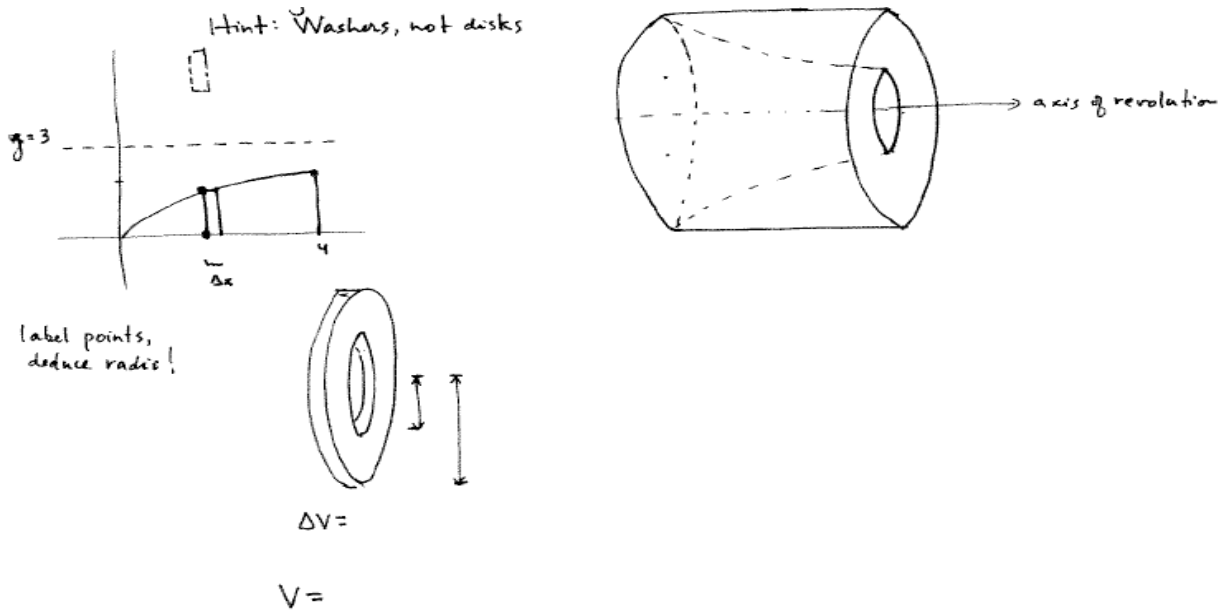
$$\Delta V \approx$$

$$V =$$

Exercise 3) Find the volume of the solid of revolution obtained by rotating the planar region bounded by  $y = \sqrt{x}$ ,  $y = 0$  (the  $x$  - axis), and the line  $x = 4$  about the  $x$  - axis.



Exercise 4) Rotate that same region about the line  $y = 3$  instead. What is the resulting solid's volume?



to be continued ...