Math 1210-001
Tuesday Apr 12
WEB L110

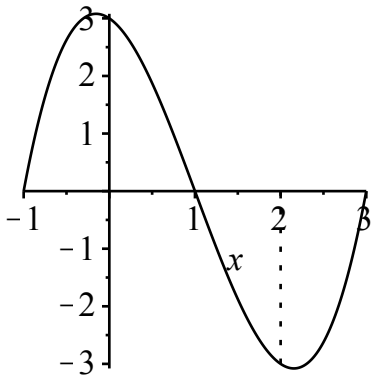
Space for warmup problem (to summarize odd/even functions, average values, from yesterday):

5.1: Areas of planar regions: We motivated our discussion of Riemann sums $\sum_{i=1}^{n} f\left(\underline{x}_i\right)\Delta x_i$ and their limits $\int_a^b f(x)\,dx$ by thinking about "signed area". Now that we're comfortable using the FTC to compute definite integrals we can find the areas of more interesting regions. That's what section 5.1 is about.

Exercise 1) Find the total area (not the signed area) between the graph of

$$y = x^3 - 3x^2 - x + 3 = (x-3)\left(x^2-1\right) = (x-3)(x-1)(x+1)$$

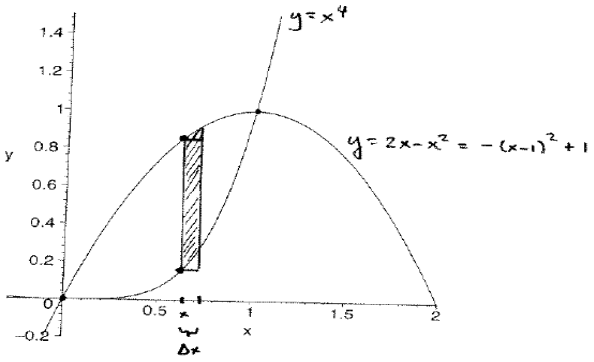
and the x-axis, on the x-interval $[-1, 2]$.



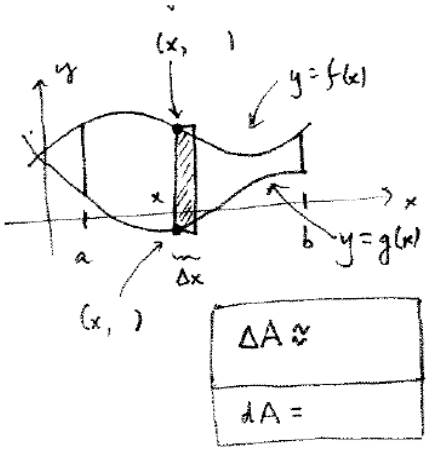
Exercise 2) Find the area of the region between the graphs $y = x^4$ and $y = 2x - x^2$.

First, identify the x- interval that the region lies above.

Then, find an expression which approximates the area ΔA of the almost rectangular region between the two graphs, with vertical sides at x and $x + \Delta x$. (Add points and information to the sketch below.) What would the corresponding Riemann sum be? What is the limit definite integral? Finally, compute the value of the definite integral to get the area of the region.



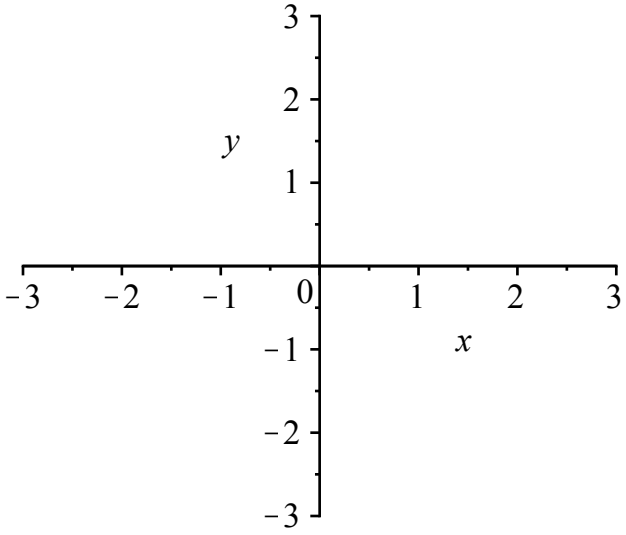
Exercise 3) Suppose $g(x) \leq f(x)$ on the interval $[a, b]$. Derive a definite integral formula for the area between the graphs $y = f(x), y = g(x)$ and the vertical lines $x = a, x = b$.



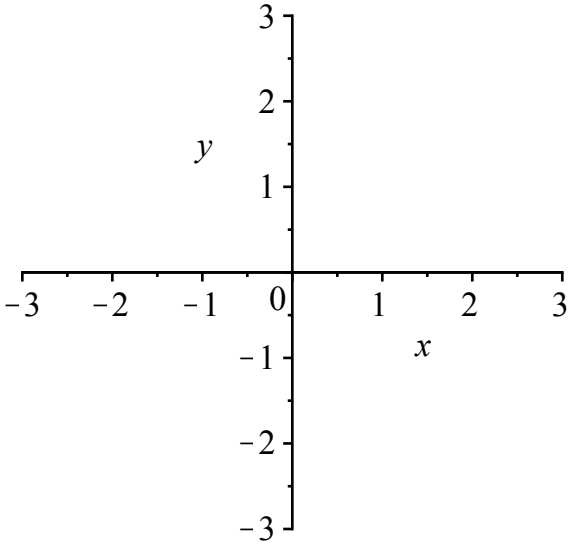
Sometimes it is better to slice horizontally than vertically!

Exercise 4) Find the area between the curves $y = x - 1$ and $x = 3 - y^2$.
Sketch and find curve intersection points first!

4a) Use vertical slicing. You will need to add two definite integrals together!



4b) Use horizontal slicing. You will only need a single *dy* integral!



Exercise 5) Suppose $h(y) \leq k(y)$ on the y-interval $[c, d]$. Find a definite integral formula for the area between the curves $x = h(y), x = k(y)$ and the horizontal lines $y = c, y = d$.