Math 5110 - Fall 2012
Mid Term Exam
Due Oct. 19, 2012, 5 pm

This takehome exam is to be done by yourself. No consulting with other people is permitted. Please give your exam to Andrew Basinski in LCB 305 or to J. Keener in LCB 314 by 5 pm Oct. 19, 2012.

For this exam you may use Maple and/or Matlab, however, unless you thoroughly describe your work, no partial credit will be given for incorrect answers. Do not turn in Maple or Matlab code.

1. The following is a model for the population dynamics of the cinnabar moth and ragwort, the perrenial plant on which the moth feeds. Since the moth lives for only one year, the model tracks the number of eggs (E) and the total ragwort biomass (B) in year n using

$$E_{n+1} = \alpha B_n, \qquad B_{n+1} = \kappa \exp(-\gamma \frac{E_n}{B_n}). \tag{1}$$

   By a simple rescaling this can be written as

$$X_{n+1} = \exp(-a \frac{Y_n}{X_n}), \qquad Y_{n+1} = X_n, \tag{2}$$

   where Y_n represents eggs, and X_n represents ragwort biomass, and $a = \gamma \alpha$.

   (a) Find all positive fixed points for this problem.
   (b) Find the Jacobian matrix for each fixed point.
   (c) For which values of a are the fixed points stable?
   (d) Simulate the behavior of this model for $a = 0.95$ starting with X at the stable fixed point and with $Y = X + 0.6$. Describe the behavior of the resulting dynamics (You should include some plots to illustrate your description). Approximately how many years does it take for X and Y to be consistently within 10% of the fixed point?

2. Suppose that a herd of cows is carefully managed. There are calves, yearlings and two year olds. Only the two year olds give birth to calves and on average they have 1.2 female calves. There is a ninety percent chance that calves survive to be yearlings and a ninety percent chance that yearlings survive to their reproductive year. After they give birth, two year old cows are sold to the butcher.

   (a) What is the reproduction matrix for this age structured population?
   (b) Is this population stable? Why or why not?
   (c) Is there a stable age distribution? If so, what is it? If not, why not?

   Suppose that a few yearlings also have calves, with .1 female calves per yearling.

   (a) What is the reproduction matrix for this new situation?
   (b) Is this population stable?
   (c) What is the stable age distribution for the herd?

(d) Suppose a rancher starts a herd with 100 calves. Simulate the population of his herd for the next twenty five years. Make a plot of the number of animals sold.

(e) Because he knows the stable age distribution, the rancher is confident that his business plan will produce a growing herd, and produce a steady income. However, when he goes to the bank to ask for a loan, the loan is not approved because the banker contends that the rancher's income will not permit him to make equal annual payments to pay off the loan. Who is correct and why?