Math 2250–016 (Underdown) Exam 1 September 25, 2014

Name ______________________________

1. [10 pts.] Solve the following nonlinear IVP explicitly for $y(x)$

$$\frac{dy}{dx} = \frac{x-5}{y^2} \qquad y(0) = 2.$$

2. [10 pts.] Find the general solution to

$$\frac{1}{x}\frac{dy}{dx} - \frac{2}{x^2}y = x\cos x, \qquad x > 0.$$

3. [20 pts.] Blood plasma is stored at 40° F. Before the plasma can be used, it must be at 90° F. When the plasma is placed in an oven at 120° F, it takes 45 min for the plasma to warm to 90°. How long will it take for the plasma to warm to 90° F if the oven temperature is set at 100° F?

   Hints: $\dfrac{dT}{dt} = k(A - T)$, here $A - T > 0$. Solve the DE symbolically, i.e. don't replace A with a number until after you have found the general solution $T(t)$.

4. [15 pts.] Use Elimination (you may use Gaussian Elimination on the augmented matrix) to find the solution set of the following linear system of equations:

$$\begin{cases} x + 2y + z = 4 & (1) \\ 3x + 8y + 7z = 20 & (2) \\ 2x + 7y + 9z = 23 & (3) \end{cases}$$

5. [15 pts.] A brine solution of salt flows at a constant rate of 4 L/min into a large tank that initially holds 100 L of water with 5 kg of salt dissolved in it. The solution inside the tank is kept well stirred and flows out of the tank at a rate of 4 L/min. If the concentration of salt in the brine entering the tank is 0.2 kg/L, determine the mass of salt in the tank after t min. Let $x(t)$ represent the amount of salt in the tank at time t, measured in kg.

6. [10 pts.] Use Euler's method to approximate a solution to the IVP:

$$\frac{dy}{dt} = t \cdot y \qquad y(0) = 2,$$

with a step size of $h = 0.5$ from $t_0 = 0$ to $t_f = 2$. Recall:

$$\boxed{\begin{aligned} \frac{dy}{dt} &= f(t,y) \qquad y(t_0) = y_0 \\ t_{n+1} &= t_n + h \\ y_{n+1} &= y_n + \underbrace{f(t_n, y_n) \cdot h}_{\Delta y} \end{aligned}}$$

Fill in the shaded blanks in the following table:

n	t	y	$f(t,y) \cdot h$	$=$	Δy
0	0	2	$(0 \cdot 2) \cdot 0.5$	$=$	0
1	0.5	2	$(0.5 \cdot 2) \cdot 0.5$	$=$	0.5
2	1.0		$(1.0 \cdot 2.5) \cdot 0.5$	$=$	1.25
3	1.5	3.75	$(1.5 \cdot 3.75) \cdot 0.5$	$=$	
4	2.0				

7. [20 pts.] True or False. Circle one.

(a) T F A solution to a differential equation must be a differentiable function.

(b) T F The general solution to a differential equation is actually an infinite family (set) of solutions.

(c) T F If an equation is separable then it is linear.

(d) T F If an equation is autonomous then it is separable.

(e) T F The equation $yy' = x$ is linear.

(f) T F The equation $y' = \sin(x)y + 2x$ is linear.

(g) T F The equation $y'' + \sin y = 0$ is linear.

(h) T F The matrix

$$\begin{bmatrix} 0 & 1 & 7 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

is in reduced row–echelon form (RREF).

(i) T F The matrix

$$\begin{bmatrix} 2 & 2 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

is in row–echelon form (REF).

(j) T F If an IVP has a solution then that solution must be unique.

8. [5 points (bonus)] Use geometry to determine a first order differential equation whose family of solution curves are concentric circles centered on the origin.

Scratch Paper

Question:	1	2	3	4	5	6	7	8	Total
Points:	10	10	20	15	15	10	20	0	100
Bonus Points:	0	0	0	0	0	0	0	5	5
Score:									