Review of 1210 Key Concepts

Continuity at a Point "we can pass the limit operator through function parenthesis."

Def Let f be defined on an open interval containing c, then f is continuous at c if $\lim_{x \to c} f(x) = f(c)$.

Derivative "The limit of a difference quotient"

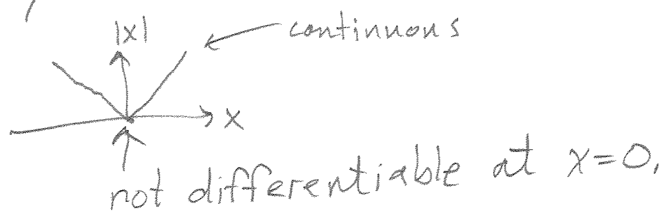
$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Q: what is the relationship between the class of differentiable functions and the class of continuous functions?

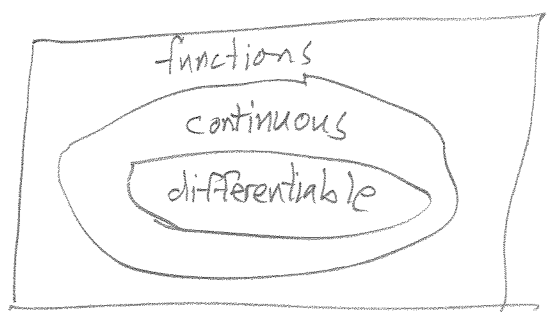
A: differentiable $\Longrightarrow$ continuous

but continuous $\not\Longrightarrow$ differentiable

Ex. $f(x) = |x|$



In Venn diagram notation:

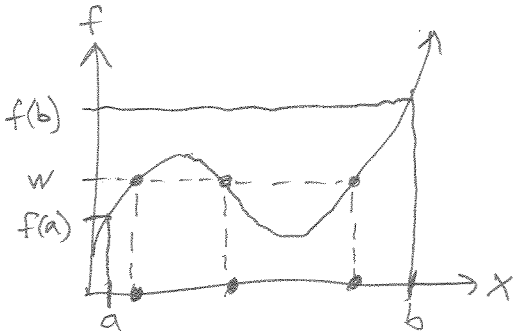


Intermediate Value Thm

Let f be defined on $[a,b]$ and let W be a number between $f(a)$ and $f(b)$.

If f is continuous on $[a,b]$
then there is at least one number c, $a<c<b$
s.t. $f(c)=W$.

Example



1st Fundamental Theorem of Calculus

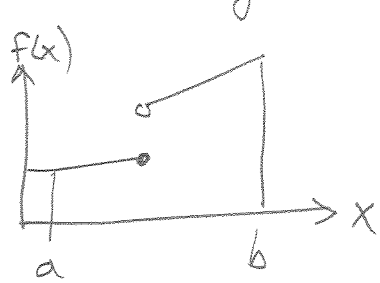
Let f be continuous on $[a,b]$ and $x \in (a,b)$,
then

$$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$$

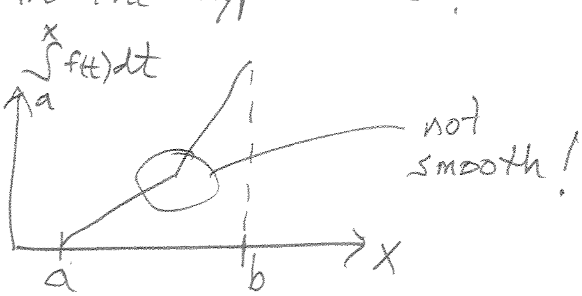
"The derivative of an accumulation function is just the 'integrand.'"

Q: what if we neglect continuity in the hypothesis?

A: Ex.



$\Longrightarrow$



f is not continuous

continuous but not differentiable!

~~Proof~~ Almost Proof of 1st FTC:

Let $F(x) = \int_a^x f(t)\,dt$, then

$$\frac{d}{dx} F(x) = \frac{d}{dx}\int_a^x f(t)\,dt$$

$$= F'(x)$$

$$= \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\int_x^{x+h} f(t)\,dt$$

Now when h is very small, $\int_x^{x+h} f(t)\,dt \approx h f(x)$ so

$$\frac{d}{dx}\int_a^x f(t)\,dt \approx \lim_{h\to 0} \frac{1}{h}\left[h f(x)\right] = f(x)$$

Thus $\frac{d}{dx} F(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

Q: why is the above "proof" not complete?

A: In a proof we obviously can't use $\approx$ (approximately equal).

To see how to resolve this, see pages 236 – 237 in your text.

2nd Fundamental Theorem of Calculus

Let f be continuous (hence integrable) on $[a,b]$, and let F be any antiderivative of f on $[a,b]$.

Then
$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

At first blush this seems rather extraordinary! Recall that we usually interpret definite integrals as area under the curve, so the left hand side of the equation above is an area. The theorem says that no matter what function you give me, as long as it has an anti-derivative, you can compute the area under the graph of the function by the difference of the antiderivative evaluated at the endpoints!

This theorem relates the two types of integration which have completely different definitions.

Name	Indefinite Integral	Definite Integral
Definition	Antiderivative	Limit of a Riemann Sum
Concept	Undoing differentiation	Area under a curve

Another version of the 2nd FTC:

$$\boxed{\int_a^b g'(x) = g(b) - g(a)}$$