§ 9.1 Infinite Sequences

Def An infinite sequence is a list of numbers separated by commas, with one number for each positive integer.

Remark We obviously can't write down such a list, so instead we write "enough" elements of the sequence to hopefully establish a "pattern", and then write "...".

Examples

1) $1, 3, 5, 7, 9, 11, 13, \ldots$ pattern: $+2$

2) $\frac{1}{5}, \frac{1}{5^2}, \frac{1}{5^3}, \frac{1}{5^4}, \ldots$ pattern: $\times\left(\frac{1}{5}\right)$

3) $-1, 1, -1, 1, -1, 1, \ldots$ pattern: $\times(-1)$

4) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots$ pattern: $\frac{1}{2n}$

5) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$ pattern: $\times\left(\frac{1}{2}\right)$

Since the elements in a list are ordered by position, a sequence can be thought of as a function from the counting numbers ($\mathbb{N}$) to the reals ($\mathbb{R}$).

$$f : \mathbb{N} \longrightarrow \mathbb{R}$$

However, we usually use f, g & h as function names for functions of a real variable, i.e. $f : \mathbb{R} \longrightarrow \mathbb{R}$.
For sequences we often use a, b or c. And instead of using "x" for the independent variable, we usually use "n".

$\longrightarrow$

Notation — real-valued function: $f(x)$ — infinite sequence: $a_n \equiv a(n)$

The subscript notation is universally recognized simply to mean a function whose domain is $N = \{1, 2, 3, 4, 5, \ldots\}$.

When possible we like to write sequences via a explicit formula, i.e, an expression in "n".

The examples on the previous page can be written:

1) $a_n = 1 + 2n$ $\quad 1, 3, 5, 7, 9, 11, 13, \ldots$

2) $b_n = \left(\frac{1}{5}\right)^n$ $\quad \frac{1}{5}, \frac{1}{5^2}, \frac{1}{5^3}, \frac{1}{5^4}, \ldots$

3) $c_n = (-1)^n$ $\quad -1, 1, -1, 1, -1, 1, \ldots$

4) $d_n = \frac{1}{2n}$ $\quad \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots$

5) $e_n = \left(\frac{1}{2}\right)^n$ $\quad \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

$\quad \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}$

What we care about when examining sequences, is whether they converge or diverge.

Def A sequence is said to converge, if the limit: $\boxed{\lim_{n \to \infty} a_n}$ exists as a finite number. Otherwise we say the sequence diverges.

<u>Note</u> You may recall from your Pre-Calculus course that when graphing rational functions you often want to check if the function has a <u>horizontal asymptote</u>. To find this you took the limit: $\boxed{\lim_{x \to \infty} f(x)}$

If this existed as a finite number, i.e. $\lim_{x \to \infty} f(x) = L$, then $y = L$ was the horizontal asymptote for the rational function. This is exactly analogous to determining whether a sequence converges:

<u>Function</u>		<u>Sequence</u>
$\lim_{x \to \infty} f(x) = L$	$\rightsquigarrow$	$\lim_{n \to \infty} a_n = L$
horizontal asymptote		convergence or divergence.

All of our previous tools for taking limits carry over to sequences. Skim section 9.1 to recall 4 theorems related to taking limits.

But honestly we don't care much about sequences, except as a tool for understanding the subject of 9.3, <u>infinite series</u>.