7.1 Basic Integration Rules

Thm U-substitution "undoes" the Chain Rule

Let $u=g(x)$ $du=g'(x)dx$

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du = F(u)+C$$
$$= F(g(x)) + C$$

where $F(u)$ is the anti-derivative of $f(u)$.

Ex. $\displaystyle\int \frac{a^{\tan t}}{\cos^2 t}\,dt$ let $u=\tan t$ $du = \sec^2 t\,dt = \frac{1}{\cos^2 t}\,dt$

$$= \int a^u\,du = \frac{1}{\ln a}\cdot a^u + c = \boxed{\frac{1}{\ln a}\cdot a^{\tan t} + C}$$

Ex. $\displaystyle\int_2^5 t\sqrt{t^2-4}\,dt$ let $u=t^2-4$ $du = 2t\,dt$ $\frac{1}{2}du = t\,dt$

$$= \frac{1}{2}\int_{2^2-4}^{5^2-4} u^{1/2}\,du = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\,u^{3/2}\Big|_0^{21} = \boxed{\frac{1}{3}(21)^{3/2}} \approx 32.08$$

34.) $\displaystyle\int \frac{1+\cos 2x}{\sin^2 2x}\,dx = \int \frac{1}{\sin^2 2x}\,dx + \int \frac{\cos 2x}{\sin^2 2x}\,dx$

$$= \int \csc^2 2x\,dx + \int (\csc 2x)\cdot(\cot 2x)\,dx$$ let $u=2x$ $\frac{1}{2}du = dx$

$$= \frac{1}{2}\int \csc^2 u\,du + \frac{1}{2}\int \csc u \cot u\,du = \boxed{\frac{-1}{2}\cot 2x + \frac{-1}{2}\csc 2x + C}$$

38.) $\int (t+1)e^{-t^2-2t-5}\,dt$

let $u = -t^2-2t-5$

$du = (-2t-2)\,dt$

$du = -2(t+1)\,dt$

$-\frac{1}{2}\,du = (t+1)\,dt$

$= \frac{-1}{2}\int e^u\,du$

$= \frac{-1}{2}e^u + C$

$= \boxed{\frac{-1}{2}e^{-t^2-2t-5} + C}$

42.) $\int \frac{5}{\sqrt{9-4x^2}}\,dx = 5\int \frac{1}{3\sqrt{1-\frac{4}{9}x^2}}\,dx$

$\sqrt{9-4x^2} = \sqrt{9(1-\frac{4}{9}x^2)}$

$= \sqrt{3^2(1-\frac{4}{9}x^2)}$

$= 3\sqrt{1-\frac{4}{9}x^2}$

$= \frac{5}{3}\int \frac{1}{\sqrt{1-\left(\frac{2x}{3}\right)^2}}\,dx$ let $u = \frac{2x}{3}$ $du = \frac{2}{3}\,dx$ $\frac{3}{2}\,du = dx$

$= \frac{5}{\cancel{3}}\cdot\frac{\cancel{3}}{2}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{5}{2}\sin^{-1}(u) + C$

$= \boxed{\frac{5}{2}\sin^{-1}\left(\frac{2}{3}x\right) + C}$

46.) $\int_0^1 \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx = \int_0^1 \frac{\sinh(2x)}{\cosh(2x)}\,dx$

let $u = \cosh(2x)$

$du = 2\cdot\sinh(2x)\,dx$

$\frac{1}{2}\,du = \sinh(2x)\,dx$

$= \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{u}\,du = \frac{1}{2}\ln|u|\Big|_{x=0}^{x=1} = \frac{1}{2}\ln(\cosh(2x))\Big|_0^1$

$= \frac{1}{2}\Big[\ln(\cosh(2)) - \underbrace{\ln(\cosh 0)}_{\ln(1)=0}\Big] = \boxed{\frac{1}{2}\ln(\cosh(2))} \approx 0.6625$