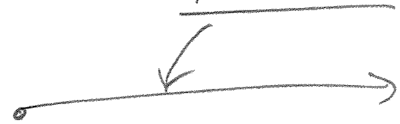
§10.5 & 10.6 Polar Coordinates

We start by signifying a point similar to the origin in Cartesian coordinates which we call the pole (or origin), and then we pick a ray (or half line) which begins at the pole and goes to infinity. This ray allows us to measure angle and distance from the origin.

Note: The polar axis is usually drawn to coincide with the x-axis.

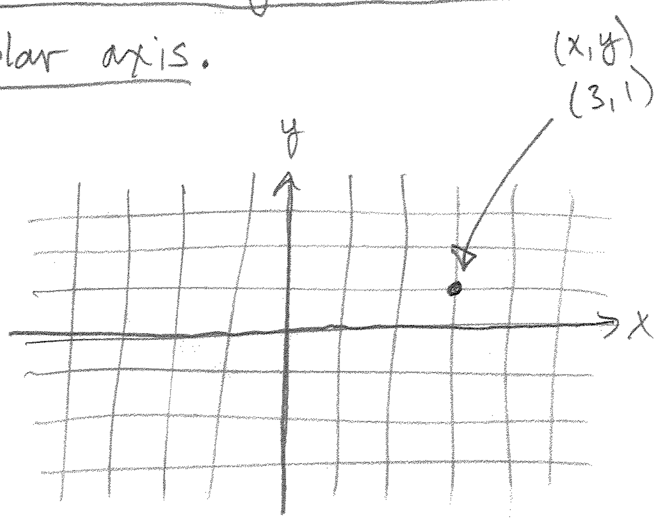
polar axis



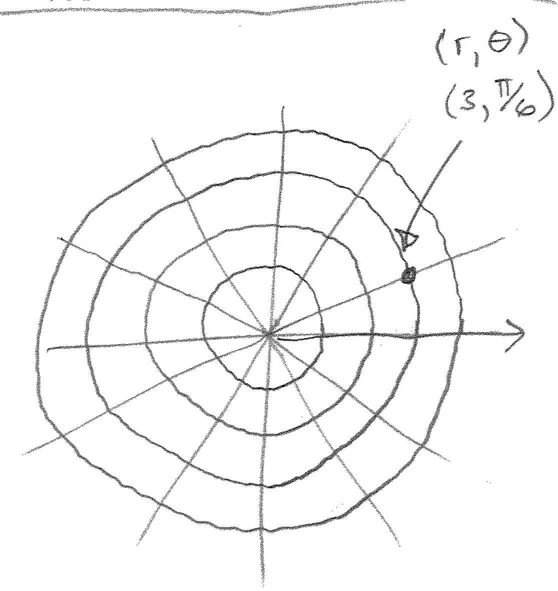
pole

Greek "theta"

Points again consist of pairs (r, θ) like (x, y), but now r measures distance from the origin, and θ measures angle in the counter-clockwise direction from the polar axis.



$\Rightarrow$

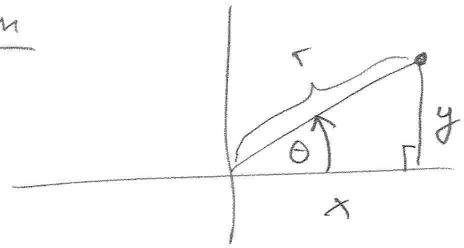


(x, y) (r, θ)

Some problems are more amenable to polar coordinates and vice versa, thus it is important to be able to go back and forth between polar and Cartesian coordinates.

Pythagorean Theorem

$r^2 = x^2 + y^2$



Trig

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

Polar to Cartesian

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

Cartesian to Polar

$$\boxed{\begin{aligned} r^2 &= x^2 + y^2 \implies r = \sqrt{x^2+y^2} \\ \tan\theta &= \frac{y}{x} \implies \theta = \arctan\left(\frac{y}{x}\right) \end{aligned}}$$

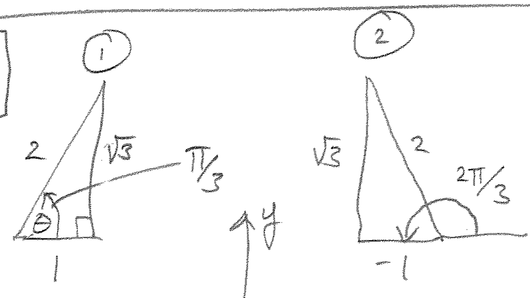
Plotting Graphs of Polar Functions / Equations

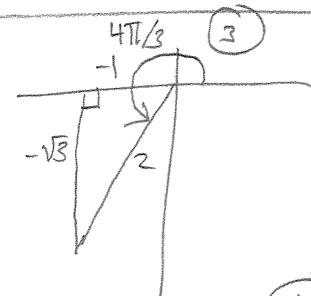
There are essentially two techniques:

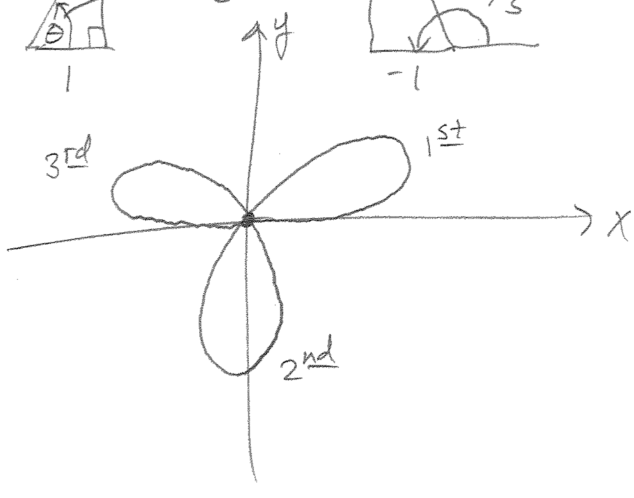
1) Create a table of (r, θ) pairs and plot them.
2) Convert to Cartesian coordinates to see if it is a recognizable curve.

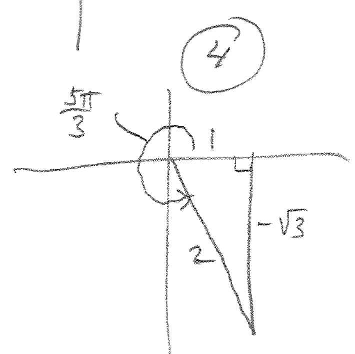
Ex Plot $\boxed{r = 2\sin(3\theta)}$

	θ	r	
	0	0	1st petal
1	$\pi/9$	$\sqrt{3}$	
	$\pi/6$	2	
2	$2\pi/9$	$\sqrt{3}$	
	$\pi/3$	0	
3	$4\pi/9$	$-\sqrt{3}$	2nd petal
	$\pi/2$	-2	
4	$5\pi/$	$\sqrt{3}$	









Ex. Show that the graph of $r = 8\sin\theta$ is a circle by converting to Cartesian coordinates.

$\boxed{r = 8\sin\theta}$ (multiply by r)

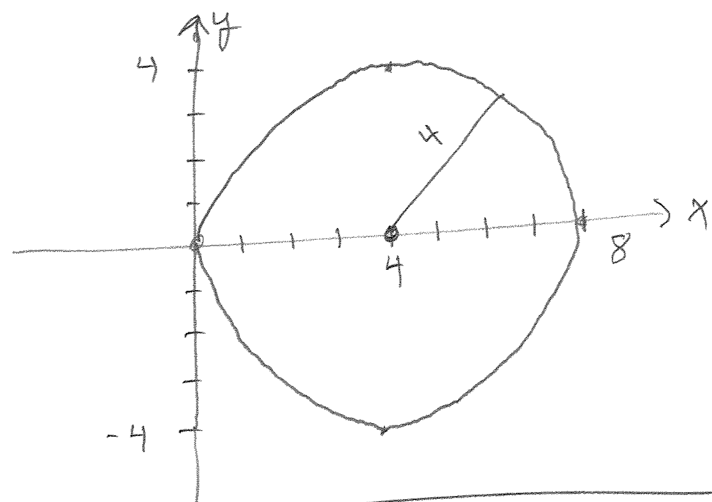
$r^2 = 8r\sin\theta$ $\quad r^2 \Rightarrow x^2+y^2$, $r\sin\theta \Rightarrow y$

$x^2+y^2 = 8y$

$x^2+y^2-8y = 0$ (complete the square)

$x^2 + (y^2-8y+16-16) = 0$

$\boxed{x^2+(y-4)^2 = 16}$ equation of a circle centered at $(0,4)$ w/ $r=4$



Cardioids and Limaçons

$\boxed{r = a \pm b\cos\theta \qquad r = a \pm b\sin\theta}$ $\quad \boxed{a, b > 0}$

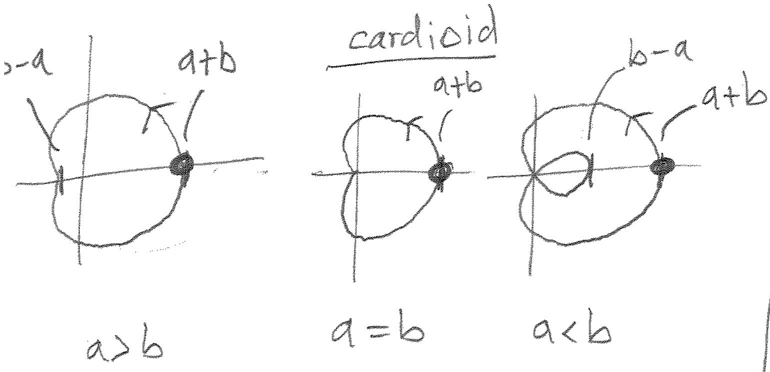
since $\cos(-\theta) = \cos(\theta)$
i.e cos is an even function
the graph of $\boxed{r = a \pm b\cos\theta}$
is symmetric with respect to the x-axis

Since $\sin(-\theta) = -\sin(\theta)$
i.e. sin is an odd function
the graph of $\boxed{r = a \pm b\sin\theta}$
is symmetric with respect to the y-axis

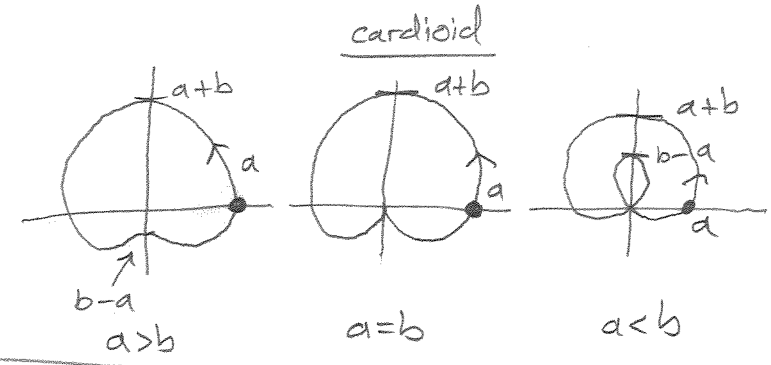
continued →

Note: The graphs of $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$ are called limaçons except when $a=b$, then they are called cardiods.

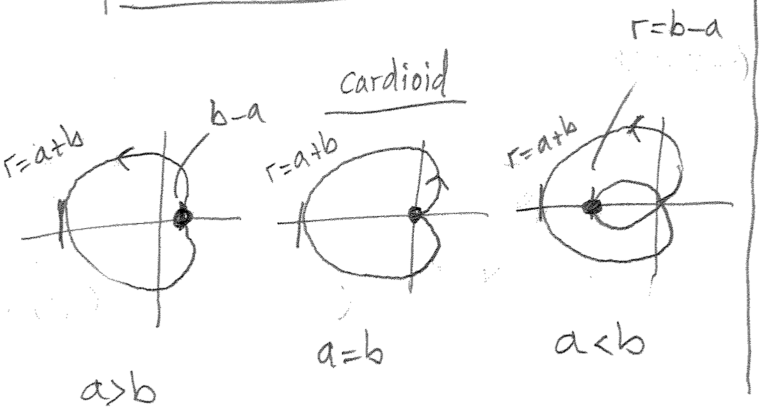
$\boxed{r = a + b\cos\theta}$



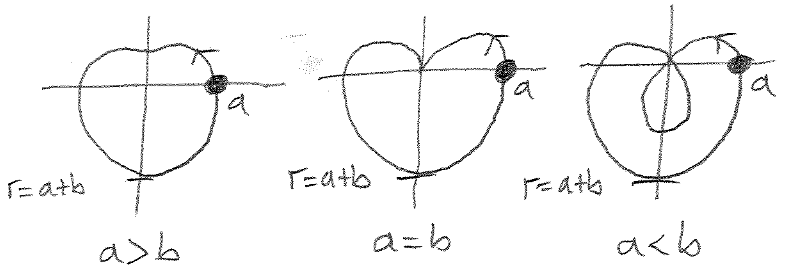
$\boxed{r = a + b\sin\theta}$



$\boxed{r = a - b\cos\theta}$



$\boxed{r = a - b\sin\theta}$



• corresponds with $\theta = 0$

Note: In the bottom two boxes $r = a+b > 0$, but remembe r can be negative, for example when $\theta = \pi$, $\cos(\theta) = -1$ and when $\theta = \frac{3\pi}{2}$, $\sin(\theta) = -1$.