Math 1220 Exam 2 February 9, 2012

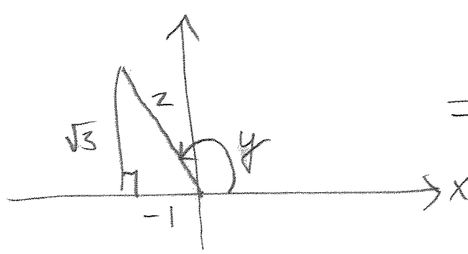
Name: Solutions

Instructions:

- Answer the questions in the space provided.
- You must show your work in order to get credit! Writing just an answer is worth 0 points, even if the answer is correct.
- Partial credit will be awarded.
- The instructor has extra scratch paper if you need it.
- Graphing and scientific calculators are allowed, but smartphones and computers are not allowed.
- This exam is closed book and closed notes, except you may use one double sided 8.5 by 11 inch page of notes.

1. Evaluate the following trigonometric expressions:

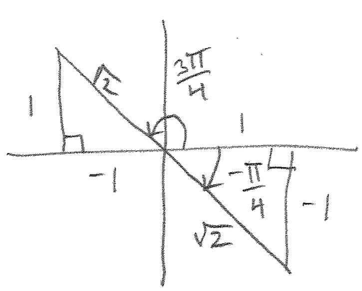
(a) [5 points] $\cos^{-1}\left(-\frac{1}{2}\right)$ $= y \iff -\frac{1}{2} = \cos y \quad y \in [0, \pi]$



$\Rightarrow \boxed{y = \frac{2\pi}{3}}$

(b) [5 points] $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ Note: $\frac{3\pi}{4} \notin (-\pi/2, \pi/2)$

which is the restricted domain of tan!



$\Rightarrow \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \boxed{-\frac{\pi}{4}}$

2. [10 points] Find $\displaystyle\int \frac{\sinh(\ln x)}{3x}\,dx.$ $= \frac{1}{3}\int \frac{\sinh(\ln x)}{x}\,dx$

let $u = \ln x$
$du = \frac{1}{x}\,dx$

$= \frac{1}{3}\int \sinh(u)\,du$

$= \frac{1}{3}\cosh(u) + C$

$= \boxed{\frac{1}{3}\cosh(\ln x) + C}$

$= \frac{1}{3}\,\frac{e^{\ln x} + e^{-\ln x}}{2} + C$

$= \frac{x + \frac{1}{x}}{6} = \boxed{\frac{x}{6} + \frac{1}{6x} + C}$

3. [10 points] Find $\displaystyle\int x^2 e^{2x}\,dx.$ $\int u\,dv = uv - \int v\,du$

$\boxed{\begin{array}{ll} u = x^2 & dv = e^{2x}dx \\ du = 2x\,dx & v = \frac{1}{2}e^{2x} \end{array}}$

$\int x^2 e^{2x}\,dx = \frac{x^2}{2}e^{2x} - \frac{2}{2}\int x e^{2x}\,dx$

$\boxed{\begin{array}{ll} u = x & dv = e^{2x}dx \\ du = dx & v = \frac{1}{2}e^{2x} \end{array}}$

$= \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{2}\int e^{2x}\,dx$

$= \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + C$

$= \boxed{\frac{e^{2x}}{2}\left(x^2 - x + \frac{1}{2}\right) + C}$

4. [10 points] Find $\int x^3 \sin(x) dx$, hint: tabular method

du	$\int dv$	+/-
x^3	$\sin x$	$-$
$3x^2$	$-\cos x$	$+$
$6x$	$-\sin x$	$-$
6	$\cos x$	$+$
0	$\sin x$	$-$

$$\int x^3 \sin x \, dx = \boxed{-x^3\cos x + 3x^2 \sin x + 6x\cos x - 6\sin x + C}$$

5. [10 points] Find $\int \sin^2(x)\cos^3(x)dx$.

$$= \int \sin^2(x)\cos^2(x)\cos(x)\,dx = \int \sin^2 x\,(1-\sin^2 x)\cos x\,dx$$

$$= \int \sin^2 x \cos x\,dx - \int \sin^4 x\cos x\,dx$$

let $u = \sin x$
$du = \cos x\,dx$

$$= \int u^2 du - \int u^4 du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$$

6. [10 points] Evaluate $\int_{-\pi}^{\pi} \sin(2x)\cos(4x)dx$.

Hint: Use symmetry.

$\sin(2x)$ is even $(+)$
$\cos(4x)$ is odd $(-)$
$(+)(-) = (-)$ odd

The integral of an odd function over an interval such as $[-\pi, \pi]$ which is symmetric with respect to the origin is zero. Hence,

$$\int_{-\pi}^{\pi} \sin(2x)\cos(4x)\,dx = \boxed{0}$$

7. [10 points] Compute $\int \frac{1}{9+x^2}dx$. $= \int \frac{1}{3^2+x^2}\,dx$

let $x = 3\tan t \Rightarrow \frac{x}{3} = \tan t \Rightarrow \tan^{-1}\left(\frac{x}{3}\right) = t$

$dx = 3\sec^2 t\,dt$

$$\int \frac{1}{9+x^2}\,dx = \int \frac{3\sec^2 t}{3^2+3^2\tan^2 t}\,dt$$

$$= \int \frac{3\sec^2 t}{3^2(1+\tan^2 t)}\,dt$$

$$= \frac{1}{3}\int \frac{\cancel{\sec^2 t}}{\cancel{\sec^2 t}}\,dt$$

$$= \frac{1}{3}t + C$$

$$= \boxed{\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C}$$

Recall:

$$\frac{\sin^2 t + \cos^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$$

$$\tan^2 t + 1 = \sec^2 t$$

8. [10 points] Use partial fractions to find $\int \frac{x+1}{x^3+x^2-6x}dx$.

$$x^3+x^2-6x = x(x^2+x-6) = x(x+3)(x-2)$$

$$\frac{x+1}{x(x+3)(x-2)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-2}$$

$$x+1 = A(x+3)(x-2) + Bx(x-2) + Cx(x+3)$$

Evaluate at the roots:

@ $x=0$: $1 = -6A \Rightarrow \boxed{A = \frac{-1}{6}}$

@ $x=-3$: $-2 = 15B \Rightarrow \boxed{B = \frac{-2}{15}}$

@ $x=2$: $3 = 10C \Rightarrow \boxed{C = \frac{3}{10}}$

Thus $\int \frac{x+1}{x(x+3)(x-2)}dx = \frac{-1}{6}\int \frac{1}{x}dx + \frac{-2}{15}\int \frac{1}{x+3}dx + \frac{3}{10}\int \frac{1}{x-2}dx$

$$= \boxed{\frac{-1}{6}\ln|x| + \frac{-2}{15}\ln|x+3| + \frac{3}{10}\ln|x-2| + C}$$

9. [10 points] Find the limit if it exists: $\lim_{x\to 0} \dfrac{\tan 3x}{\ln(1+4x)}$ $\sim \dfrac{0}{0}$

$$\lim_{x\to 0} \frac{\tan 3x}{\ln(1+4x)} \overset{H}{=} \lim_{x\to 0} \frac{3\sec^2(3x)}{4/(1+4x)} = \lim_{x\to 0} \frac{3\cancel{(1+4x)}^{\to 1}}{4\cancel{\cos^2(3x)}_{\to 1}} = \boxed{\frac{3}{4}}$$

rewrite only

10. [10 points] Find the limit if it exists: $\lim_{t\to\infty} \left(\dfrac{t-1}{t+1}\right)^t$

Let $y = \left(\frac{t-1}{t+1}\right)^t$ then $\ln y = \underbrace{t}_{\to\infty} \underbrace{\ln\left(\frac{t-1}{t+1}\right)}_{\to 0} \sim \infty\cdot 0$

$$\ln y = \frac{\ln\left(\frac{t-1}{t+1}\right)}{\frac{1}{t}} \sim \frac{0}{0}$$

$$D_x\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$$

$= 2$

$$\lim_{t\to\infty} \ln y = \lim_{t\to\infty} \frac{\ln\left(\frac{t-1}{t+1}\right)}{\frac{1}{t}} \overset{H}{=} \lim_{t\to\infty} \frac{\left(\frac{t+1}{t-1}\right)\cdot\frac{(t+1)-(t-1)}{(t+1)^2}}{\frac{-1}{t^2}}$$

$$= \lim_{t\to\infty} \frac{-2t^2}{(t-1)(t+1)} = \lim_{t\to\infty} \frac{-2t^2}{t^2-1} = -2$$

$$\Rightarrow \lim_{t\to\infty} y = \boxed{e^{-2}}$$

11. [0 points (bonus)] When does the narwhal bacon?

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	0	
Total:	100	