# Math1090 Practice Final Exam

1. Find the inverse, $f^{-1}(x)$ for each given function and state the domain of the both the function and its inverse.

(a) $f(x)=\ln(x-1)$ Domain of $f(x)$: $x-1>0 \iff \boxed{x>1}$

Use the "pants" technique: Let $h(x)=\ln(x)$ $g(x)=x-1$

then $f(x)=h\circ g(x)$. To find $f^{-1}$ we must first undo $h(x)$ with $h^{-1}(x)$ and then undo $g(x)$ with $g^{-1}(x)$:

$h^{-1}(x)=e^x$ $g^{-1}(x)=x+1$ $\Rightarrow$ $f^{-1}(x)=g^{-1}\circ h^{-1}(x)=g^{-1}(h^{-1}(x))$

$= g^{-1}(e^x) = e^x+1$

$f^{-1}(x)$ = $e^x+1$

Domain of $f^{-1}(x)$ : $\mathbb{R}$

Domain of $f(x)$ : $x>1$

(b) $f(x)=\frac{x^5}{x^5-1}$ Here we can use the "formal algebraic" technique: $y=\frac{x^5}{x^5-1} \Rightarrow x=\frac{y^5}{y^5-1}$ Now solve for $y$:

$x(y^5-1)=y^5 \Rightarrow xy^5-x=y^5 \Rightarrow xy^5-y^5=x$

$\Rightarrow (x-1)y^5=x \Rightarrow y^5=\frac{x}{x-1} \Rightarrow \boxed{y=\sqrt[5]{\frac{x}{x-1}}=f^{-1}(x)}$

Domain of $f^{-1}(x)$: $x-1\neq 0 \Rightarrow x\neq 1$

$\parallel$ $\parallel$ $f(x)$:

$x^5-1\neq 0$

$x^5\neq 1$

$x\neq \sqrt[5]{1}=1$

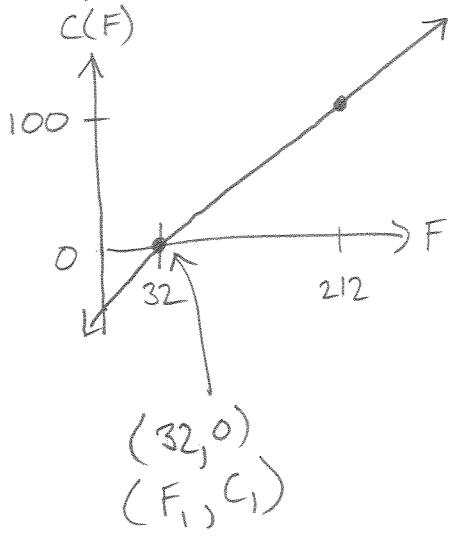
$x\neq 1$

$f^{-1}(x)$ = $\sqrt[5]{\frac{x}{x-1}}$

Domain of $f^{-1}(x)$ : $x\neq 1$ OR $(-\infty,1)\cup(1,\infty)$

Domain of $f(x)$ : $x\neq 1$ OR " $\cup$ "

2. Find the function which relates Celsius to Fahrenheit using the fact that 32 degrees Fahrenheit corresponds with 0 degrees Celsius, and 212 degrees Fahrenheit corresponds to 100 degrees Celsius.




$$m = \frac{\text{rise}}{\text{run}} = \frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$$

$$y - y_1 = m(x - x_1)$$

$$C - C_1 = m(F - F_1)$$

$$C - 0 = \frac{5}{9}(F - 32)$$

$$\boxed{C = \frac{5}{9}(F - 32)}$$

3. For an investment that earns 6% interest compounded monthly, how much should be deposited at the beginning of each month in order to have $250,000 after 20 years?

Use the FVAD (Future Value of an Annuity Due) formula to get the "Sinking Fund" formula for an Annuity Due by solving for R!

FVAD

$$S_{due} = (1 + r_c)S = (1 + r_c)\left(\frac{R\left((1+r_c)^N - 1\right)}{r_c}\right)$$

$$R = \frac{S_{due}}{(1+r_c)}\left(\frac{r_c}{\left((1+r_c)^N - 1\right)}\right) = \frac{250{,}000}{(1.005)}\left(\frac{.005}{(1.005)^{240} - 1}\right)$$

$$\boxed{R = 538.39}$$

$S_{due} = 250{,}000$

$N = n \cdot t = 12 \cdot 20 = 240$

$r_c = \frac{r}{n} = \frac{.06}{12} = .005$

Monthly deposit: $538.39

4. Showing all your steps clearly, solve this system of linear equations.

$$2x-4y+2z=-4$$
$$4x-9y+7z=2$$
$$-2x+4y-3z=10$$

$\Updownarrow$

$$\left[\begin{array}{ccc|c} 2 & -4 & 2 & -4 \\ 4 & -9 & 7 & 2 \\ -2 & 4 & -3 & 10 \end{array}\right] + R_1 \qquad \left[\begin{array}{ccc|c} -4 & 8 & -4 & 8 \\ 2 & -4 & 2 & -4 \\ 4 & -9 & 7 & 2 \\ 0 & 0 & -1 & 6 \end{array}\right] - 2R_1$$

$$\left[\begin{array}{ccc|c} 2 & -4 & 2 & -4 \\ 0 & -1 & 3 & 10 \\ 0 & 0 & -1 & 6 \end{array}\right] \begin{matrix} (\tfrac{1}{2}) \\ (-1) \\ (-1) \end{matrix} \qquad \left[\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & 1 & -6 \end{array}\right]$$

$\Updownarrow$

Now use back substitution:

$$x - 2y + z = -2 \quad (1)$$
$$y - 3z = -10 \quad (2)$$
$$\boxed{z = -6} \quad (3)$$

(3) & (2) $\Rightarrow$ $y - 3(-6) = -10$

$y + 18 = -10$

$\boxed{y = -28}$ (*)

(1) & (*) & (3) $\Rightarrow$ $x - 2(-28) + (-6) = -2$

$x + 56 - 6 = -2$

$\boxed{x = -52}$

Solution: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -52 \\ -28 \\ -6 \end{bmatrix}$

5. Find the vertex, axis of symmetry and zeros of the parabola

$y=-(x-3)^2+6$

Recall $y=a(x-h)^2+k$ is vertex form

So $(h,k) = (3,6) = \text{vertex} \Rightarrow$ axis of symmetry $x=3$

zeros: $0 = -(x-3)^2+6$

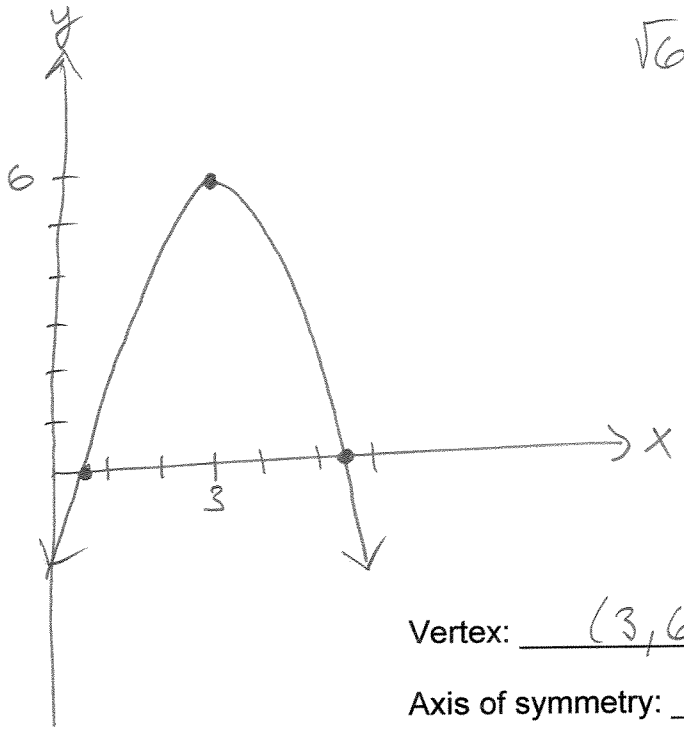
$\Rightarrow (x-3)^2 = 6$

$\Rightarrow x-3 = \pm\sqrt{6}$

$\Rightarrow x = 3 \pm \sqrt{6}$

$\sqrt{6} \approx 2.45$

Graph:




Vertex: (3,6)

Axis of symmetry: $x=3$

Zeros: $x = 3 \pm \sqrt{6}$

6. Let $f(x)=3x^2+1$ , $g(x)=x+6$ , and $h(x)=\frac{x}{x^3-10}$ . Find the following.

(a) $(fg)(x) = (3x^2+1)(x+6)$

$= 3x^3 + 18x^2 + x + 6$

$(fg)(x) =$ $3x^3+18x^2+x+6$

(b) $(g\circ h)(x) = g(h(x))$

$= g\left(\frac{x}{x^3-10}\right)$

$= \frac{x}{x^3-10} + 6 = \frac{x}{x^3-10} + \frac{6(x^3-10)}{x^3-10}$

$= \frac{6x^3+x-60}{x^3-10}$

$(g\circ h)(x) =$ $\frac{6x^3+x-60}{x^3-10}$

(c) $(f\circ g)(1)$

$(f\circ g)(x) = f(g(x)) = f(x+6)$

$= 3(x+6)^2+1$

$= 3(x^2+12x+36)+1 = 3x^2+36x+109$

$(f\circ g)(1) = 3\cdot 1^2 + 36\cdot 1 + 109 =$ $148$

(d) $(f+g)(x)$

$(f+g)(x) = (3x^2+1) + (x+6)$

$= 3x^2 + x + 7$

$(f+g)(x) =$ $3x^2+x+7$

7. You are buying your first home. You have found a home that costs $190,000. You have been able to secure a 30-year loan from a bank at an interest rate of 5.35% compounded monthly.

(a) What will your monthly payment be?

$$R = S\left(\frac{r_c}{1-(1+r_c)^{-N}}\right)$$

$S = 190{,}000$

$N = n \cdot t = 12 \cdot 30 = 360$

$r_c = \frac{r}{n} = \frac{.0535}{12} = .0044583\overline{3}$

$$R = 190{,}000\left(\frac{.0044583\overline{3}}{1-(1.0044583\overline{3})^{-360}}\right)$$

$R = 1{,}060.99$

Monthly payment: $1,060.99

(b) How much will you pay in interest over the life of the loan?

$\text{Interest} = NR - S$

$= 360(1{,}060.99) - 190{,}000$

$= 191{,}956.40$

Total interest paid: $191,956.40

8. Graph the solution set of the following system of inequalities and find and label all vertices of the boundary.

① $3x+4y\geq 12$
② $x-y\geq 2$
③ $x\leq 6$
④ $y\geq 0$

① $4y \geq -3x+12 \Rightarrow \boxed{y \geq -\frac{3}{4}x+3}$

② $-y \geq -x+2 \Rightarrow \boxed{y \leq x-2}$

$\left.\begin{array}{l} 3x+4y=12 \\ x-y=2 \end{array}\right\} \Rightarrow \begin{array}{r} 3x+4y=12 \\ -3x+3y=-6 \\ \hline 7y=6 \end{array} \Rightarrow \boxed{y=\frac{6}{7}}$

$x-y=2$
$x-\frac{6}{7}=2$
$x=\frac{14}{7}+\frac{6}{7}=\frac{20}{7}$
$\boxed{x=\frac{20}{7}=2\frac{6}{7}}$