5.2 Simple Interest, Compound Interest and APY

Arithmetic Sequence

recursive $a_n = a_{n-1} + d$

iterative $a_n = a_1 + (n-1)d$

simple interest $S = P + t(Pr) = P(1+rt)$

} linear

Geometric Sequence

recursive $a_n = d\,a_{n-1}$

iterative $a_n = a_1 d^{n-1}$

compound interest $S = P\left(1+\frac{r}{n}\right)^{nt}$

} exponential

$S = S(t)$ = value of account after t years.

P = principal or the initial investment ($P = a_1$)

r = interest rate

n = # of compounding periods per year

Def. interest accrued/earned is defined to be

$$\boxed{I = S - P}$$

Ex 1 Invest \$10,000 at 7.5% for four years.

a) simple interest

$S = 10{,}000(1+(0.075)4)$

$= 10{,}000(1.3)$

$= 13{,}000$

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b) compound interest (compounded twice yearly $n=2$)

$S = 10{,}000\left(1+\frac{0.075}{2}\right)^{2\cdot 4}$

$= 10{,}000\,(1.0375)^8$

$= 13{,}424.71$

Ex 2 Mary borrowed $3,000 at an interest rate of 18%. How much interest will accrue after 65 weeks?

a) **simple interest**

$$65 \text{ weeks}\left(\frac{1 \text{ yr}}{52 \text{ weeks}}\right) = 1.25 \text{ yr.} = t$$

$$I = S - P = P(1+rt) - P$$

$$= P + Prt - P$$

$$= Prt$$

$$= (3{,}000)(0.18)(1.25)$$

$$= \boxed{\$675}$$

b) **compounded quarterly interest**

$$I = S - P = P\left(1 + \frac{r}{n}\right)^{nt} - P$$

$$= 3{,}000\left(1 + \frac{0.18}{4}\right)^{4\cdot(1.25)} - 3{,}000$$

$$= 3{,}000(1.045)^5 - 3{,}000$$

$$= 3{,}738.55 - 3{,}000.00$$

$$= \boxed{\$\,738.55}$$

Ex 4 How much do you need to invest now in order to retire with $2,000,000 in 40 years if interest compounds monthly at 8%? (Ans. P = $82,394.77)

$$S = P\left(1 + \frac{r}{n}\right)^{nt}$$

Annual Percentage Yield (APY)

APY is a tool that allows you to compare the return from different interest bearing accounts. It allows you to compare apples to apples.

$$APY = \left(1 + \frac{r}{n}\right)^n - 1$$ periodic compounding

$$APY = e^r - 1$$ continuous compounding

Ex. 6 Which is the best investment deal in the long run?

a) 10% simple interest

b) 9% compounded annually

c) 8.75% " quarterly

d) 8.6% " continuously