4.2 Exponential Functions

Until now we have focused on functions that involve the input "x" being raised to some power, e.g. x^2 or x^3 or $x^{\frac{1}{2}} = \sqrt{x}$ or $x^{\frac{1}{3}} = \sqrt[3]{x}$, and combinations thereof.

Now we will examine functions which reverse this, i.e. functions where the input is found in the <u>power</u> instead of in the <u>base</u> e.g.

power or exponent

$$\boxed{f(x) = a^x} \quad \text{where } a \in \mathbb{R},\ a>0,\ a \neq 1$$

base (constant)

<u>Examples</u>: $f(x) = 2^x$ $\qquad h(x) = \pi^x$

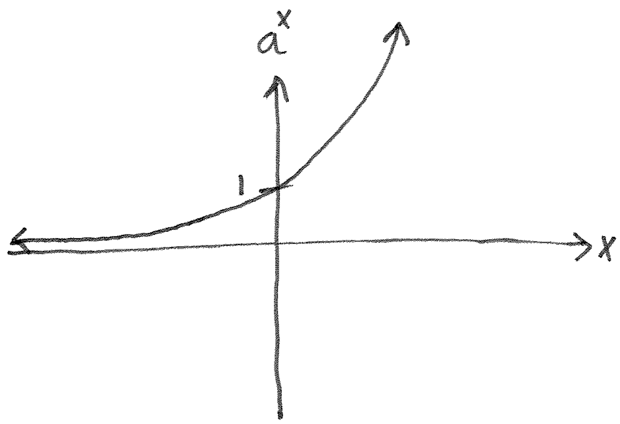
$g(x) = \left(\frac{1}{2}\right)^x$ $\qquad k(x) = e^x$

where $e \approx 2.718218$ is a number like π because it is irrational, i.e. can't be written as a ratio or fraction. In other words, the decimal part of e is infinite or never ends. "e" is called the <u>natural base</u>.

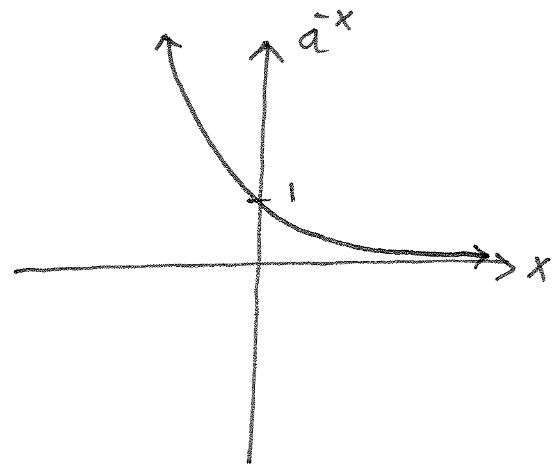
Notice: $\frac{1}{2} = 2^{-1} \Rightarrow \left(\frac{1}{2}\right)^x = \left(2^{-1}\right)^x = 2^{-1\cdot x} = 2^{-x}$

So $\boxed{a^{-x} = \left(\frac{1}{a}\right)^x}$

There are two basic graphs to remember!



exponential <u>growth</u>



exponential <u>decay</u>

Suppose $\boxed{a=2}$, then if $f(x) = 2^x$

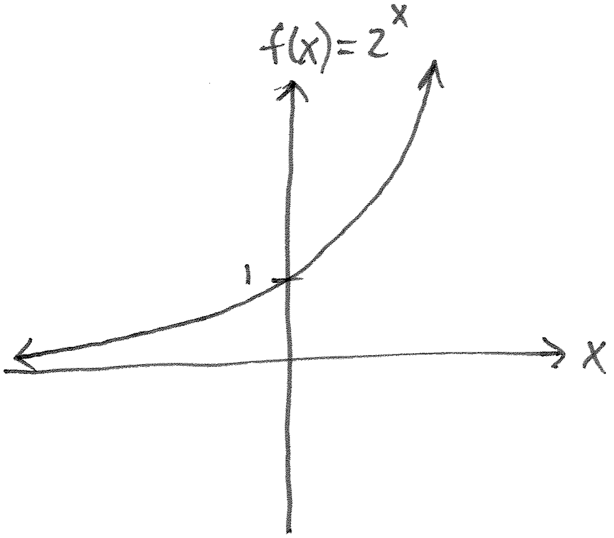
x	f(x)
-2	$2^{-2} = (2^{-1})^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$

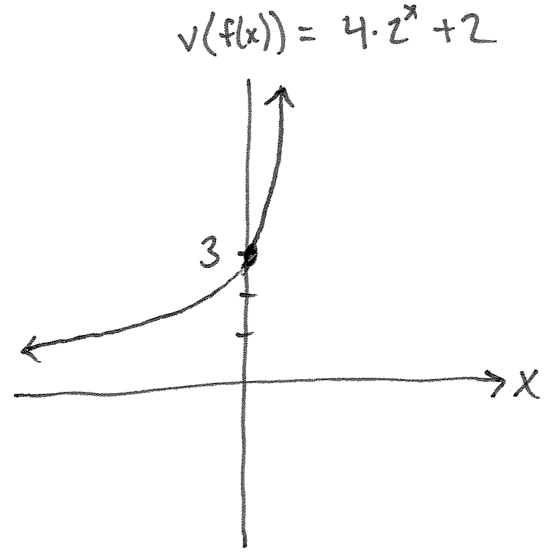
We can compose exponential functions with ~~of~~ other functions. For example let $\boxed{v(x) = 4x+2}$ and $\boxed{f(x) = 2^x}$ then $v \circ f(x) = v(f(x)) =$

$\to = v(2^x) = 4 \cdot 2^x + 2$

vertical stretch ↑ ↑ shift up by 2

→





Exponential functions are often used to model populations:

Ex 4 (from the book)

The population of Bricktown has been modeled to grow according to the function:

$$P = 135{,}000\,(1.012)^t \qquad t = \text{years after } 2002$$

a) what was the population in 2005?

b) what will be the population in 2012?

Solution

a) $2005 - 2002 = 3$ So set $t=3$

$\Rightarrow P(3) = 135{,}000\underbrace{(1.012)^3}$

to compute this on your calculator, type 1.012 $\boxed{x^y}$ 3 $\boxed{=}$ OR

1.012 $\boxed{\wedge}$ 3 $\boxed{=}$

$\Rightarrow P(3) = 135{,}000\,(1.036433728)$

$\boxed{\cong 139{,}919}$

b) @ 2012: $2012 - 2002 = 10 = t$

$P(10) = 135{,}000\,(1.012)^{10}$

$\boxed{\cong 152{,}103}$