Quick Review

3.1 Quadratic Equations

Four methods for solving quadratic equations: $ax^2+bx+c=0$

1. Square root technique – Only works when $b=0$.
2. Factoring – Doesn't always work.
3. Completing the square – Always works – what real men do.
4. Quadratic formula – Always works $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

3.2 Functions – Domain and Range

Ex. $g(x) = \dfrac{\sqrt{x+5}}{\sqrt{5-x}}$

We have to ensure two things:

1) Don't take even roots of negative numbers.
2) Denominator is never zero.

Numerator: $x+5 \geq 0 \Rightarrow \boxed{x \geq -5}$ AND

Denominator: $5-x>0 \Rightarrow -x>-5 \Rightarrow \boxed{x<5}$

Domain: $\boxed{-5 \leq x < 5}$ Range: $\mathbb{R}$

3.3 Parabolas – The Graphs of Quadratic Equations

$ax^2+bx+c \Rightarrow a(x-h)^2+k$

Recall: The point (h,k) is the vertex of the parabola.

$h = \dfrac{-b}{2a}$ $k = \dfrac{4ac-b^2}{4a}$

3.) Find the number of units that need to be produced and sold to break even given the revenue and cost functions.

$R(x) = 100\sqrt{x}$

$C(x) = 2x + 100$

Recall: $P(x) = R(x) - C(x)$ and the break even point is when $P(x) = 0$, so:

$$P(x) = R(x) - C(x)$$
$$= 100\sqrt{x} - (2x + 100)$$
$$P(x) = -2x + 100\sqrt{x} - 100 = 0$$
$$x - 50\sqrt{x} + 50 = 0$$

This doesn't look like the standard quadratic eqn., but it still is one:

$$x - 50\sqrt{x} + 50 = (\sqrt{x})^2 - 50\sqrt{x} + 50 = 0$$

Use the quadratic formula? Pshaw!

$$\begin{array}{r} \overset{1}{2}5 \\ \times\ 25 \\ \hline 125 \\ +500 \\ \hline 625 \end{array}$$

$$\left[(\sqrt{x})^2 - 50\sqrt{x} + 625\right] - 625 + 50 = 0$$
$$(\sqrt{x} - 25)^2 = 575$$
$$\sqrt{x} - 25 = \pm\sqrt{575}$$
$$\sqrt{x} = 25 \pm \sqrt{575}$$
$$x = (25 \pm \sqrt{575})^2$$

$x \approx 1.04$ or

$x \approx 2398.96$

9.) Given the supply and demand equations, find the equilibrium price and quantity.

supply : $p = q^2 + 10$

demand : $p = -3q^2 - 6q + 86$

Solve: $q^2 + 10 = -3q^2 - 6q + 86$

$\Rightarrow \quad 4q^2 + 6q - 76 = 0$

$2q^2 + 3q - 38 = 0$ irreducible

$q^2 + \frac{3}{2}q - 19 = 0$ $\qquad \left(\frac{3}{2} \cdot \frac{1}{2}\right)^2 = \frac{9}{16}$

$\left(q^2 + \frac{3}{2}q + \frac{9}{16}\right) - \frac{9}{16} - 19 = 0$

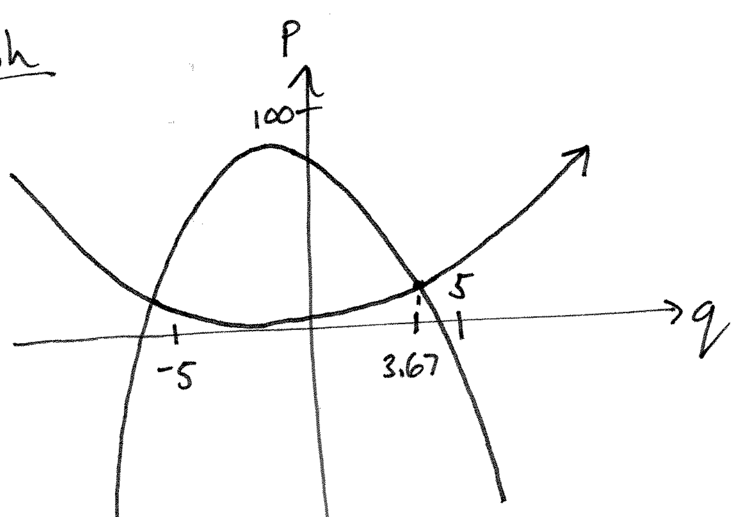
$\left(q + \frac{3}{4}\right)^2 = \frac{9}{16} + \frac{304}{16} = \frac{313}{16}$

$q + \frac{3}{4} = \pm\sqrt{\frac{313}{16}}$

$\boxed{q = \frac{-3}{4} \pm \frac{\sqrt{313}}{4}}$ $\qquad q \approx -5.17$ OR

$q \approx 3.67$

Graph



17.) A company has daily production costs of

$$C(x) = 800 - 10x + 0.25x^2$$

How many items produced each day yield a minimum cost? What is the minimum cost?

$$\min = \text{vertex} = (h, k)$$

$$= \left(\frac{-b}{2a}, \frac{-b^2 + 4ac}{4a} \right)$$

$$\frac{-b}{2a} = \frac{-(-10)}{2\left(\frac{1}{4}\right)} = \frac{10}{\frac{1}{2}} = 20 \implies 20 \text{ items/day}$$

$$\frac{-b^2 + 4ac}{4a} = \frac{-(-10)^2 + 4\left(\frac{1}{4}\right)800}{4\left(\frac{1}{4}\right)}$$

$$= \frac{-100 + 800}{1}$$

$$= 700 \implies \$700/\text{day is the minimum daily cost.}$$