MATH 4200-1 FALL 2008

Second Mock Exam

INSTRUCTOR: H.-PING HUANG

LAST NAME ______________________________

FIRST NAME ______________________________

ID NO. ______________________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1	25	______
PROBLEM 2	25	______
PROBLEM 3	25	______
PROBLEM 4	25	______
PROBLEM 5	25	______
TOTAL	125	______

PROBLEM 1

(25 pt) Find a path γ which traces once around the triangle with vertices $0, 1, i$ in the counter-clockwise direction, starting at 0. For this path γ, find $\int_\gamma \overline{z} dz$.

Is it generally true that

$$\overline{\int_\gamma f(z) dz} = \int_\gamma \overline{f(z)} dz?$$

PROBLEM 2

(25 pt) Use Cauchy's formula to show that

$$\int_{|z-1|=1} \frac{1}{z^2-1} dz = \pi i, \quad \int_{|z+1|=1} \frac{1}{z^2-1} dz = -\pi i.$$

Hint: use a partial fraction decomposition of the integral.

PROBLEM 3

(25 pt) Prove that if f is analytic on the disc $D_R(z_0)$ and $|f(z)| \leq M$ on $D_R(z_0)$, the $|f'(z_0)| \leq M/R$.

Suppose $p(z) = a_3z^3 + a_2z^2 + a_1z + a_0$ is a polynomial of degree 3. If $|p(z)| \leq 1$ on the unit circle $\{z : |z| = 1\}$, then show that $|a_3| \leq 6$.

PROBLEM 4

(25 pt) Prove that if f is an entire function which satisfies $|f(z)| \geq 1$ on the entire plane, then f is constant.

Prove that if an entire function has real part which is bounded above, then the function is constant.

PROBLEM 5

(25 pt) Show that if f is a non-constant analytic function on a connected open set U and if f has no zeroes on U, then there are no points of U where $|f(z)|$ has a local minimum.

Show that if f is a non-constant, continuous function on $\overline{D_1}(0)$, which is analytic on $D_1(0)$ and $|f(z)| = 1$ for all z on the unit circle, then f has a zero somewhere in $D_1(0)$.