Name ___________________________ Class Time _____

Math 2280 Extra Credit Problems
Chapter 6
S2019

Submitted work. Please submit one stapled package per chapter. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number, e.g., Xc6.1-4. Please attach this printed sheet to simplify your work.

Problem Xc6.1-4. (Phase Portraits)

Find the equilibrium points for the system. Plot a phase diagram using the `maple` code below.

$$\begin{cases} \dfrac{dx}{dt} &= x - 2y + 3, \\ \dfrac{dy}{dt} &= x - y + 2. \end{cases}$$

Example: Plot the phase diagram of $\mathbf{u}' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \mathbf{u} + \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ using `maple`.

```
with(DEtools):
equilEQ:=[0=x+2*y+4,0=3*y+5];
solve(equilEQ,{x,y});# find diagram center (a,b)
a:=-2/3;b:=-5/3;
de:=[diff(x(t),t)=x(t)+2*y(t)+4,diff(y(t),t)=3*y(t)+5];
ic:=[[x(0)=0,y(0)=-1],[x(0)=-1,y(0)=-1.5],[x(0)=0.5,y(0)=-2],
[x(0)=0.5,y(0)=-1.5],[x(0)=-0.7,y(0)=-1.7]];
DEplot(de,[x(t),y(t)],t=-10..10,ic,x=a-2..a+2,y=b-2..b+2,stepsize=0.05);
```

Problem Xc6.1-8. (Equilibrium Points)

Find the equilibrium points for the system. Plot a phase diagram. The graph window should include the three equilibrium points.

$$\begin{cases} \dfrac{dx}{dt} &= x - 2y, \\ \dfrac{dy}{dt} &= 4x - x^3. \end{cases}$$

Problem Xc6.1-18. (Stability)

Determine if the equilibrium point $(0, 0)$ is stable, asymptotically stable, or unstable. Identify the equilibrium point as a node, saddle, center or spiral by examination of its computer-generated direction field.

(a) $x' = y$, $y' = -x$

(b) $x' = y$, $y' = -5x - 4y$

(c) $x = -2x$, $y' = -2y$

(d) $x = y$, $y' = x$

Problem Xc6.2-2. (Classification by Eigenvalues)

Compute the eigenvalues of A. Determine stability of equilibrium $(0, 0)$ and classify as node (proper/improper), saddle, spiral, center.

(a) $A \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(b) $A\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(c) $A\begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$

(c) $A\begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}$

Problem Xc6.2-12. **(Phase Portrait)**

Find the equilibrium point (it is unique) and plot by computer a phase diagram.

$$\begin{cases} \dfrac{dx}{dt} & = & x + y - 7, \\ \dfrac{dy}{dt} & = & 3x - y - 5. \end{cases}$$

Problem Xc6.2-22. **(Almost Linear System)**

Linearize the system at its equilibria and determine the stability and type of each. Plot a phase diagram by computer to verify the claims made.

$$\begin{cases} \dfrac{dx}{dt} & = & 2x - 5y + x^3, \\ \dfrac{dy}{dt} & = & 4x - 6y + y^4. \end{cases}$$

Problem Xc6.3-8. **(Predator-Prey System)**

Linearize the system at equilibrium point $(0, 0)$. Verify that the phase diagram of the nonlinear system at $(0, 0)$ is a saddle.

$$\begin{cases} \dfrac{dx}{dt} & = & x(5 - x - y), \\ \dfrac{dy}{dt} & = & y(-2 + x). \end{cases}$$

Problem Xc6.3-9. **(Predator-Prey System)**

Linearize the system at equilibrium point $(5, 0)$. Verify that the phase diagram of the nonlinear system at $(5, 0)$ is a saddle.

$$\begin{cases} \dfrac{dx}{dt} & = & x(5 - x - y), \\ \dfrac{dy}{dt} & = & y(-2 + x). \end{cases}$$

Problem Xc6.3-10. **(Predator-Prey System)**

Linearize the system at equilibrium point $(2, 3)$. Verify that the phase diagram of the nonlinear system at $(2, 3)$ is an asymptotically stable spiral.

$$\begin{cases} \dfrac{dx}{dt} & = & x(5 - x - y), \\ \dfrac{dy}{dt} & = & y(-2 + x). \end{cases}$$

Problem Xc6.4-4. **(Almost Linear System)**

Linearize at $(0, 0)$ and classify the equilibrium point $(0, 0)$ of the nonlinear system, using a phase diagram to verify the conclusion.

$$\begin{cases} \dfrac{dx}{dt} & = & 2\sin x + \sin y, \\ \dfrac{dy}{dt} & = & \sin x + 2\sin y. \end{cases}$$

Problem Xc6.4-8. **(Almost Linear System)**

Linearize at all equilibria and classify the equilibrium points of the nonlinear system. Use a phase diagram to verify the conclusions.

$$\begin{cases} \dfrac{dx}{dt} &= y, \\ \dfrac{dy}{dt} &= \sin \pi x - y. \end{cases}$$

End of extra credit problems chapter 6.