Name ______________________________ Class Time ______

Math 2280 Extra Credit Problems
Chapter 4
S2019

Submitted work. Please submit one stapled package per chapter. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number, e.g., Xc4.1-8. Please attach this printed sheet to simplify your work.

Problem XcL3.1. (Numerical Solutions)

You may submit this problem only for score increases on numerical solutions 2.4 to 2.6.

Solve symbolically by chapter 1 methods the initial value problem $y' = 2xy^2$, $y(0) = 1$. Do an answer check in maple or by hand. Answer: $y = 1/(1 - x^2)$. This problem has no numerical work!

Problem XcL3.2. (Numerical Solutions)

You may submit this problem only for score increases on numerical solutions 2.4 to 2.6. This problem counts as three (3) problems.

Solve $y' = 2xy^2$, $y(0) = 1$ numerically for the value of $y(0.5)$ using (1) Euler's method, (2) Heun's method and (3) the RK4 method. Use step size $h = 0.1$. Include computer code and a print of the data. Report the answers in a table for x-values 0, 0.1, 0.2, 0.3, 0, 4, 0.5.

Problem XcL4.1. (Numerical Solutions)

You may submit this problem only for score increases on numerical solutions 2.4 to 2.6.

Solve symbolically by chapter 1 methods the initial value problem $y' = e^{-y}$, $y(0) = 0$. Do an answer check in maple or by hand. Answer: $y = \ln(1 + x)$. This problem has no numerical work!

Problem XcL4.2. (Numerical Solutions)

You may submit this problem only for score increases on numerical solutions 2.4 to 2.6. This problem counts as three (3) problems.

Solve $y' = e^{-y}$, $y(0) = 0$ numerically for the value of $y(1.0)$ using (1) Euler's method, (2) Heun's method and (3) the RK4 method. Use step size $h = 0.001$. Include a computer code appendix in the report, but do not print the data. Report the answers in a table for x-values 0, 0.2, 0, 4, 0.6, 0.8, 1.0. Include the percentage error $E = 100|\ln(2) - y(1.0)|/|\ln(2)|$ in your report, one error report for each of the three methods.

Problem Xc4.1-8. (Transform to a first order system)

Use the position-velocity substitution $u_1 = x(t)$, $u_2 = x'(t)$, $u_3 = y(t)$, $u_4 = y'(t)$ to transform the system below into vector-matrix form $\mathbf{u}'(t) = A\mathbf{u}(t)$. Do not attempt to solve the system.

$$x'' - 2x' + 5y = 0, \quad y'' + 2y' - 5x = 0.$$

Problem Xc4.1-20a. (Dynamical systems)

Prove this result for system

(1)
$$\begin{array}{rcrcr} x' & = & ax & + & by, \\ y' & = & cx & + & dy. \end{array}$$

Theorem. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and define $\mathbf{trace}(A) = a + d$. Then $p_1 = -\mathbf{trace}(A)$, $p_2 = \det(A)$ are the coefficients in the determinant expansion

$$\det(A - rI) = r^2 + p_1 r + p_2$$

and $x(t)$ and $y(t)$ in equation (**??**) are both solutions of the differential equation $u'' + p_1 u' + p_2 u = 0$.

Problem xC4.1-20b. **(Solve dynamical systems)**

(a) Apply the previous problem to solve

$$\begin{array}{rcrcr} x' & = & 2x & - & y, \\ y' & = & x & + & 2y. \end{array}$$

(b) Use first order methods to solve the system

$$\begin{array}{rcrcr} x' & = & 2x & - & y, \\ y' & = & & & 2y. \end{array}$$

Problem Xc4.2-12. **(General solution answer check)**

(a) Verify that $\mathbf{x}_1(t) = e^{3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\mathbf{x}_2(t) = e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ are solutions of $\mathbf{x}' = A\mathbf{x}$, where

$$A = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}.$$

(b) Apply the Wronskian test $\det(\mathbf{aug}(\mathbf{x}_1, \mathbf{x}2)) \neq 0$ to verify that the two solutions are independent.

(c) Display the general solution of $\mathbf{x}' = A\mathbf{x}$.

End of extra credit problems chapter 4.