Name ____________________________ Class Time _____

Math 2280 Extra Credit Problems
Chapter 3
S2019

Submitted work. Please submit one stapled package per chapter. Kindly label problems $\boxed{\textbf{Extra Credit}}$. Label each problem with its corresponding problem number, e.g., $\boxed{\text{XC3.2-18}}$. Please attach this printed sheet to simplify your work.

Problem XcL2.1. (maple lab 2)

You may submit this problem only for score increases on maple lab 2.

Consider the linear differential equation $u' + ku = ka(t)$, $u(0) = u_0$, where $a(t) = 1 + \sin(\pi(t-3)/12)$. Solve the equation for $u(t)$ and check your answer in maple. Use maple assist for integration.

Problem XcL2.2. (maple lab 2)

You may submit this problem only for score increases on maple lab 2.

Consider the linear differential equation $u' + ku = ka(t)$, $u(0) = u_0$, where $a(t) = 1 + \sin(\pi(t-3)/12)$. Find the steady-state periodic solution of this equation and check your answer in maple.

Problem XC3.1-all. (Second order DE)

This problem counts as full credit for 5.1, if 5.1 was not submitted, and 100 otherwise. Solve the following seven parts.

(a) $y'' + 4y' = 0$ (b) $4y'' + 12y' + 9y = 0$ (c) $y'' + 2y' + 5y = 0$
(d) $21y'' + 10y' + y = 0$ (e) $16y'' + 8y' + y = 0$ (f) $y'' + 4y' + (4+\pi)y = 0$

(g) Find the differential equation $ay'' + by' + cy = 0$, if e^{-x} and e^{x} are solutions.

Problem XC3.2-18. (Initial value problems)

Given $x^3y''' + 6x^2y'' + 4xy' - 4y = 0$ has three solutions x, $1/x^2$, $\dfrac{\ln|x|}{x^2}$, prove by the Wronskian test that they are independent and then solve for the unique solution satisfying $y(1) = 1$, $y'(1) = 5$, $y''(1) = -11$.

Problem XC3.2-22. (Initial value problem)

Solve the problem $y'' - 4y = 2x$, $y(0) = 2$, $y'(0) = -1/2$, given a particular solution $y_p(x) = -x/2$.

Problem XC3.3-8. (Complex roots)

Solve $y'' - 6y' + 25y = 0$.

Problem XC3.3-10. (Higher order complex roots)

Solve $y^{iv} + \pi^2 y''' = 0$.

Problem XC3.3-16. (Fourth order DE)

Solve the fourth order homogeneous equation whose characteristic equation is $(r-1)(r^3-1) = 0$.

Problem XC3.3-32. (Theory of equations)

Solve $y^{iv} - y''' + y'' - 3y' - 6y = 0$. Use the theory of equations [factor theorem, root theorem, rational root theorem, division algorithm] to completely factor the characteristic equation. You may check answers by computer, but please display the complete details of factorization.

Problem XC3.4-20. (Overdamped, critically damped, underdamped)

(a) Consider $2x''(t) + 12x'(t) + 50x(t) = 0$. Classify as overdamped, critically damped or underdamped.

(b) Solve $2x''(t) + 12x'(t) + 50x(t) = 0$, $x(0) = 0$, $x'(0) = -8$. Express the answer in the form $x(t) = C_1 e^{\alpha_1 t}\cos(\beta_1 t - \theta_1)$.

(c) Solve the zero damping problem $2u''(t) + 50u(t) = 0$, $u(0) = 0$, $u'(0) = -8$. Express the answer in phase-amplitude form $u(t) = C_2 \cos(\beta_2 t - \theta_2)$.

(d) Using computer assist, display on one graphic plots of $x(t)$ and $u(t)$. The plot should showcase the damping effects. A hand-made replica of a computer or calculator graphic“ is sufficient.

Problem XC3.4-34. **(Inverse problem)**

A body weighing 100 pounds undergoes damped oscillation in a spring-mass system. Assume the differential equation is $mx''+cx'+kx = 0$, with t in seconds and $x(t)$ in feet. Observations give $x(0.4) = 6.1/12$, $x'(0.4) = 0$ and $x(1.2) = 1.4/12$, $x'(1.2) = 0$ as successive maxima of $x(t)$. Then $m = 3.125$ slugs. Find c and k.

Atoms. An **atom** is a term of the form $x^k e^{ax}$, $x^k e^{ax} \cos bx$ or $x^k e^{ax} \sin bx$. The symbol k is a non-negative integer. Symbols a and b are real numbers with $b > 0$. In particular, 1, x, x^2, e^x, $\cos x$, $\sin x$ are atoms. Any distinct list of atoms is linearly independent.

Roots and Atoms. Define **atomRoot**(A) as follows. Symbols α, β, r are real numbers, $\beta > 0$ and k is a non-negative integer.

atom A	**atomRoot**(A)
$x^k e^{rx}$	r
$x^k e^{\alpha x} \cos \beta x$	$\alpha + i\beta$
$x^k e^{\alpha x} \sin \beta x$	$\alpha + i\beta$

The fixup rule for undetermined coefficients can be stated as follows:

> *Compute* **atomRoot**(A) *for all atoms A in the trial solution. Assume r is a root of the characteristic equation of multiplicity k. Search the trial solution for atoms B with* **atomRoot**$(B) = r$*, and multiply each such B by x^k. Repeat for all roots of the characteristic equation.*

Problem Xc3.5-1A. **(AtomRoot Part 1)**

1. Evaluate **atomRoot**(A) for $A = 1$, x, x^2, e^{-x}, $\cos 2x$, $\sin 3x$, $x \cos \pi x$, $e^{-x} \sin 3x$, x^3, e^{2x}, $\cos x/2$, $\sin 4x$, $x^2 \cos x$, $e^{3x} \sin 2x$.

2. Let $A = xe^{-2x}$ and $B = x^2 e^{-2x}$. Verify that **atomRoot**$(A) =$ **atomRoot**(B).

Problem Xc3.5-1B. **(AtomRoot Part 2)**

3. Let $A = xe^{-2x}$ and $B = x^2 e^{2x}$. Verify that **atomRoot**$(A) \neq$ **atomRoot**(B).

4. Atoms A and B are said to be **related** if and only if the derivative lists A, A', ... and B, B', ... share a common atom. Prove: atoms A and B are related if and only if **atomRoot**$(A) =$ **atomRoot**(B).

Problem XC3.5-6. **(Undetermined coefficients, fixup rule)**

Find a particular solution $y_p(x)$ for the equation $y^{iv} - 4y'' + 4y = xe^{2x} + x^2 e^{-2x}$. Check your answer in `maple`.

Problem XC3.5-12. ()

Find a particular solution $y_p(x)$ for the equation $y^{iv} - 5y'' + 4y = xe^x + x^2 e^{2x} + \cos x$. Check your answer in `maple`.

Problem XC3.5-22. **(Fixup rule, trial solution)**

Report a trial solution y for the calculation of y_p by the method of undetermined coefficients, after the fixup rule has been applied. To save time, do not do any further undetermined coefficients steps.

$$y^v + 2y''' + 2y'' = 5x^3 + e^{-x} + 4 \cos x.$$

Hint: Test $r^2(r^3 + 2r + 2) = 0$ when $r =$ **atomRoot**(B) and B is an atom in the initial trial solution. This means a test only for $r = 0, -1, i$.

Problem XC3.5-54. (Variation of parameters)

Solve by variation of parameters for $y_p(x)$ in the equation $y'' - 16y = xe^{4x}$. Check your answer in `maple`.

Problem XC3.5-58. (Variation of parameters)

Solve by the method of variation of parameters for $y_p(x)$ in the equation $(x^2 - 1)y'' - 2xy' + 2y = x^2 - 1$. Use the fact that $\{x, 1 + x^2\}$ is a basis of the solution space of the homogeneous equation. Apply (33) in the textbook, after division of the leading coefficient $(x^2 - 1)$. Check your answer in `maple`.

Problem XC3.6-4. (Harmonic superposition)

Write the general solution $x(t)$ as the superposition of two harmonic oscillations of frequencies 2 and 3, for the initial value problem $x''(t) + 4x(t) = 16 \sin 3t$, $x(0) = 0$, $x'(0) = 0$.

Problem XC3.6-8. (Steady-state periodic solution)

The equation $x''(t) + 3x'(t) + 3x(t) = 8 \cos 10t + 6 \sin 10t$ has a unique steady-state periodic solution of period $2\pi/10$. Find it.

Problem XC3.6-18. (Practical resonance)

Use the equation $\omega = \sqrt{\dfrac{k}{m} - \dfrac{c^2}{2m^2}}$ to decide upon practical resonance for the equation $mx'' + cx' + kx = F_0 \cos \omega t$ when $F_0 = 10$, $m = 1$, $c = 4$, $k = 5$. Sketch the graph of $C(\omega) = \dfrac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$ and mark on the graph the location of the resonant frequency (if any). See Figure 5.6.9 in Edwards-Penney.

Problem XC3.7-4. (LR-circuit)

An LR-circuit with emf $E(t) = 100e^{-12t}$, inductor $L = 2$, resistor $R = 40$ is initialized with $i(0) = 0$. Find the current $i(t)$ for $t \ge 0$ and argue physically and mathematically why the observed current at $t = \infty$ should be zero.

Problem XC3.7-12. (Steady-state of an RLC-circuit)

Compute the steady-state current in an RLC-circuit with parameters $L = 5$, $R = 50$, $C = 1/200$ and emf $E(t) = 30 \cos 100t + 40 \sin 100t$. Report the amplitude, phase-lag and period of this solution.

End of extra credit problems chapter 3.