MATH 2280-001, REVIEW PROBLEMS FOR EXAM 2

Note. Exam 2 is on Wednesday, November 11th. It will cover the course material that we have covered in class since exam 1, that is sections 3.4,3.5,3.6,4.1,5.1,5.2,5.4 (just eigenvalues of multiplicity 2),6.1 and 5.3.You will be allowed one side of an 8.5 by 11 inch sheet of notes. Calculators will not be allowed. The following are some review problems. This should not be considered a "practice exam" (I have not yet checked it for appropriate length or difficulty), but the problems are of the sort that you should be able to do. I will have solutions posted by noon on Tuesday, possibly earlier.

1. Consider the mass-spring equation

$$mx'' + cx' + kx = f(t)$$

a. Explain what each term represents (in terms of forces).

b. If $m = 0.25$, $c = 0$, $k = 4$, and $f(t) = 0$ for all t, find the motion given the initial conditions $x(0) = 1$ and $x'(0) = 4$.

c. What is the amplitude and period of motion? Express the solution as $x(t) = A\cos(\omega t - \phi)$.

d. If in addition, $f(t) = \cos(\alpha t)$, for what value of α would there be resonance?

e. If in addition to the values given in b for m,k and $f(t)$, if $c = 1$ would the motion be underdamped, overdamped or critically damped?

f. If $f(t) = \cos(t) + e^{-t}\sin(t)$, what form would a particular solution x_p take? Assume the values of m,c, and k are as in part e. You don't have to find x_p.

2. Let A be the matrix:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

a. Find the eigenvalues of A.

b. Find 3 linearly independent solutions of $\mathbf{x}' = A\mathbf{x}$. What is the general solution?

3. Draw the phase diagram for the following linear system, and determine whether $(0, 0)$ (the origin) is stable or unstable, and whether it is a center, a spiral, a saddle or a node.

$$x' = -x - y$$
$$y' = x - y$$

4. Suppose that tank 1, and tank 2, have mixtures of a solute in a solution, of volumes 100 gallons and 50 gallons respectively, and that the solution from tank 1 flows through a tube into tank 2 at r=10 gallons a minute, and from tank 2 into tank 1 through another tube at the same rate. Let $x(t)$ and $y(t)$ denote the amounts of solute in tanks 1 and 2 respectively.

a. Show that x and y satisfy the system:

$$\begin{aligned} x' &= -\frac{1}{10}x + \frac{1}{5}y \\ y' &= \frac{1}{10}x - \frac{1}{5}y \end{aligned}$$

b. Use the eigenvector method to find the general solution of the system in part a.

c. If initially (at $t = 0$), there are x_0 ponds of solute in tank 1, and no solute in tank 2, find $x(t)$ and $y(t)$. What are these values as $t \to \infty$?