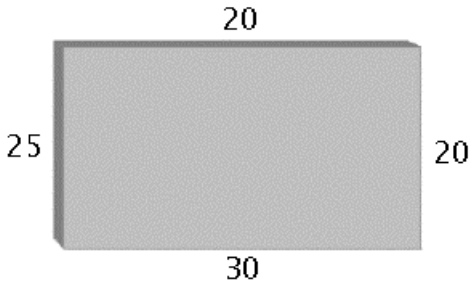
Enrichment. Mean Value Property.

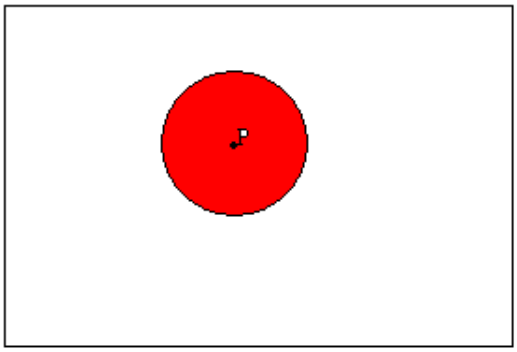
Sample Problem 8. Heat Transfer and the Mean Value Property.

Consider the cross section of a long rectangular dam on a river, represented in the figure.



The boundaries of the dam are subject to three factors: the temperature in degrees Celsius of the air (20), the water (25), and the ground at its base (30).

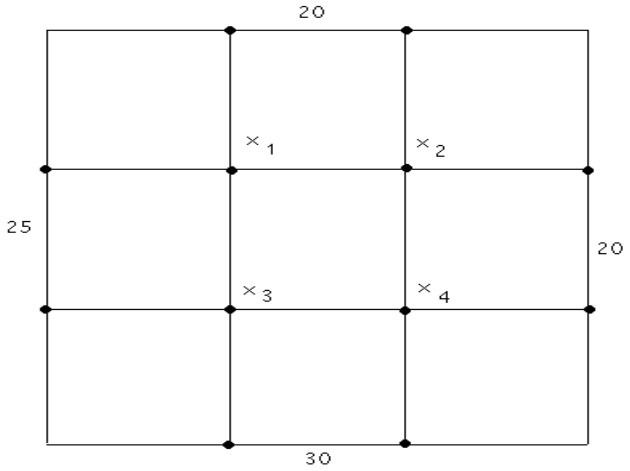
An analysis of the heat transfer from the three sources will be done from the equilibrium temperature, which is found by the Mean Value Property below.



The Mean Value Property

If a plate is at thermal equilibrium, and C is a circle contained in the plate with center P, then the temperature at P is the average value of the temperature function over C.

A version of the Mean Value Property says that the temperature at center P of circle C is the average of the temperatures at four equally-spaced points on C. We construct a grid as in the figure below, label the unknown temperatures at interior grid points as x_1, x_2, x_3, x_4, then use the property to obtain four equations.



Four-Point Temperature Averages

$$x_1 = \frac{1}{4}(20 + 25 + x_2 + x_3)$$

$$x_2 = \frac{1}{4}(20 + 20 + x_1 + x_4)$$

$$x_3 = \frac{1}{4}(25 + 30 + x_1 + x_4)$$

$$x_4 = \frac{1}{4}(20 + 30 + x_2 + x_3)$$

Solve the equations for the four temperatures $x_1 = 23.125, x_2 = 21.875, x_3 = 25.625, x_4 = 24.375$ by any method.

References. EPH Chapters 12, 13, on heat transfer. Used in Partial Differential Equations 3150. Intro Differential Equations 2280 uses Chapter 9 of a different Edwards-Penney textbook.