Chapters 1,2: Sample Problem 7

The velocity of a crossbow arrow fired upward from the ground is given at different times in the following table.

Time t in seconds	Velocity $v(t)$ in ft/sec	Location
0.000	50	Ground
1.413	0	Maximum
2.980	-45	Near Ground Impact

(a) The velocity $v(t)$ can be approximated by a quadratic polynomial

$$z(t) = at^2 + bt + c$$

which reproduces the table data. Find three equations for the coefficients a, b, c. Then solve for them to obtain $a \approx 2.238$, $b \approx -38.55$, $c = 50$.

(b) Assume a linear drag model $v' = -32 - \rho v$. Substitute the polynomial answer $v = z(t)$ of (a) into this differential equation, then substitute $t = 0$ and solve for $\rho \approx 0.131$.

(c) Solve the model $w' = -32 - \rho w$, $w(0) = 50$ to get $w(t) = -\frac{32}{\rho} + \left(50 + \frac{32}{\rho}\right) e^{-\rho t}$. Substitute $\rho = 0.131$. Then $w(t) = -244.2748092 + 294.2748092\, e^{-0.131\, t}$ is an exponential model for linear drag which might reproduce the crossbow data.

(d) Compare $w(t)$ and $z(t)$ in a plot. Comment on the plot and what it means. Bear in mind that $w(t)$ is an exponential model while $z(t)$ is a quadratic model. Neither of them are the true velocty $v(t)$ which produced the crossbow data.

References. Edwards-Penney sections 2.3, 3.1, 3.2. Course document on **Linear algebraic equations**:

`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/linearequDRAFT.pdf`

Course document on **Newton kinematics**:

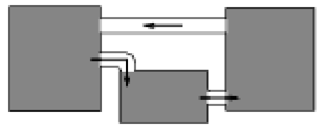
`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/newtonModelsDE2008.pdf`

Chapters 1,2. Sample Problem 8

Consider the system of differential equations

$$\begin{aligned} x_1' &= -\tfrac{1}{6}x_1 & & + \tfrac{1}{6}x_3, \\ x_2' &= \tfrac{1}{6}x_1 - \tfrac{1}{3}x_2, & & \\ x_3' &= & \tfrac{1}{3}x_2 & - \tfrac{1}{6}x_3, \end{aligned}$$

for the amounts x_1, x_2, x_3 of salt in recirculating brine tanks, as in the figure:



Recirculating Brine Tanks A, B, C

The volumes are $60, 30, 60$ for A, B, C, respectively.

The steady-state salt amounts in the three tanks are found by formally setting $x_1' = x_2' = x_3' = 0$ and then solving for the symbols x_1, x_2, x_3. Solve the corresponding linear system of algebraic equations to obtain the answer $x_1 = x_3 = 2c$, $x_2 = c$, which means the total amount of salt is uniformly distributed in the tanks in ratio $2 : 1 : 2$.

References. Edwards-Penney sections 3.1, 3.2, 7.3 Figure 5. Course document on **Linear algebraic equations**:

`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/linearequDRAFT.pdf`

Course document on **Systems and Brine Tanks**:

`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/systemsBrineTank.pdf`