Chapters 1,2. Sample Problem 7.

(a) Let $t_1 = 1.413$, $t_2 = 2.98$. Use $at^2+bt+c = v(t)$ for $t=0, t_1, t_2$ to obtain the system

$$\begin{cases} a\cdot 0^2 + b\cdot 0 + c = 50 \\ a\cdot t_1^2 + b\cdot t_1 + c = 0 \\ a\cdot t_2^2 + b\,t_2 + c = -45 \end{cases}$$

Then $\boxed{c=50}$. The 3X3 system reduces to a 2x2 system

$$\begin{cases} a\,t_1^2 + b\,t_1 = -50 \\ a\,t_2^2 + b\,t_2 = -95 \end{cases}$$

$$\begin{cases} a + b/t_1 = -50/t_1^2 \\ a\,t_2^2 + b\,t_2 = -95 \end{cases} \qquad \text{mult}(1, 1/t_1^2)$$

$$\begin{cases} a + b/t_1 = -50\,t_1^2 \\ 0 + b\cdot t_3 = -95 + \dfrac{50\,t_2^2}{t_1^2} \end{cases} \qquad \text{combo}(1,2,-t_2^2) \text{ where } t_3 = t_2 - \frac{t_2^2}{t_1}$$

Then $\boxed{b} = \dfrac{1}{t_2-t_1}\left(\dfrac{-50t_2}{t_1} + 95\,\dfrac{t_1}{t_2}\right) = \boxed{-38.54760463}$

$\boxed{a} = \dfrac{1}{t_2-t_1}\left(\dfrac{50}{t_1} - \dfrac{95}{t_2}\right) = \boxed{2.23772148}$

The example gives evidence for why technology is used on systems of equations. For 2-digit accuracy, it is less hand work, and quite fast with a calculator.

(b) Substitution gives $2at + b = -32 - \rho(at^2+bt+c)$, Then $t=0$ implies $b = -32 - \rho c$. Calculator gives $\rho = (-32-b)/c = 0.1309520926 \cong 0.131$

(c) $w = \text{equil Sol} + \dfrac{c_1}{\text{integ factor}} = \dfrac{-32}{\rho} + \dfrac{c_1}{e^{\rho t}}$. Then $w(0)=50$ implies $c_1 = 50 + 32/\rho$.

(d) A good plot is $|v(t) - w(t)|$ on $0 \le t \le 3$. It shows max error of 0.3. typo: v(t) above should be z(t)

Chapters 1,2. Sample Problem 8.

System $\begin{cases} -\frac{1}{6}x_1 + \frac{1}{6}x_3 = 0 \\ \frac{1}{6}x_1 - \frac{1}{3}x_2 = 0 \\ \frac{1}{3}x_2 - \frac{1}{6}x_3 = 0 \end{cases}$ has augmented matrix equal to

$$\left(\begin{array}{ccc|c} -\frac{1}{6} & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & -\frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & -\frac{1}{6} & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right) \quad \text{mult}(1,6), \text{mult}(2,6), \text{mult}(3,6)$$

$$\left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right) \quad \text{combo}(1,2,1)$$

$$\left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \text{Combo}(2,3,1)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \text{mult}(1,-1), \text{mult}(2,-1)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \text{mult}(2, \tfrac{1}{2}) \qquad \text{Last frame}$$

$$\begin{cases} x_1 - x_3 = 0 \\ x_2 - \frac{1}{2}x_3 = 0 \\ 0 = 0 \end{cases}$$

reduced echelon system
last frame algorithm applies

Answer

$$\begin{cases} x_1 = t_1 \\ x_2 = \frac{1}{2}t_1 \\ x_3 = t_1 \end{cases} \qquad -\infty < t_1 < \infty$$

Let $t_1 = 2c$
Then

$$\begin{cases} x_1 = 2c \\ x_2 = c \\ x_3 = 2c \end{cases}$$