Chapter 2. Sample Problem 5. A graphic called a **phase diagram** displays the behavior of all solutions of $u' = F(u)$. A **phase line diagram** is an abbreviation for a direction field on the vertical axis (u-axis). It consists of equilibrium points and signs of $F(u)$ between equilibria. A phase diagram can be created solely from a phase line diagram, using just three drawing rules:

1. Solutions don't cross.

2. Equilibrium solutions are horizontal lines $u = c$. All other solutions are increasing or decreasing.

3. A solution curve can be moved rigidly left or right to create another solution curve.

Use these tools on the equation $u' = u(u^2 - 4)$ to make a phase line diagram, and then make a phase diagram with at least 8 threaded solutions. Label the equilibria as stable, unstable, funnel, spout, node.

References. Edwards-Penney section 2.2. Course document on **Stability**:
`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/2250phaseline.pdf`

Chapter 2. Sample Problem 6. An autonomous differential equation $\frac{dy}{dx} = F(x)$ with initial condition $y(0) = y_0$ has a formal solution

$$y(x) = y_0 + \int_0^x F(u)du.$$

The integral may not be solvable by calculus methods. In this case, the integral is evaluated numerically to compute $y(x)$ or to plot a graphic. There are three basic numerical methods that apply, the rectangular rule (RECT), the trapezoidal rule (TRAP)and Simpson's rule (SIMP).

Apply the three methods for $F(x) = \sin(x^2)$ and $y_0 = 0$ using step size $h = 0.2$ from $x = 0$ to $x = 1$. Then fill in the blanks in the following table. Use technology if it saves time. Lastly, compare the four data sets in a plot, using technology.

$x-$values	0.0	0.2	0.4	0.6	0.8	1.0
$y-$ to 10 digits	0.0	0.0026663619	0.02129435557	0.07133622797	0.1657380596	0.3102683017
$y-$RECT values	0.0	0.0	0.007997866838			0.2297554431
$y-$TRAP values	0.0		0.02392968750	0.07508893150		0.3139025416
$y-$SIMP values	0.0	0.002666288917	0.02129368017		0.1657330636	

References. Edwards-Penney Sections 2.4, 2.5, 2.6, because methods Euler, Modified Euler and RK4 reduce to RECT, TRAP, SIMP methods when $f(x, y)$ is independent of y, i.e., an equation $y' = F(x)$. Course document on numerical solution of $y' = F(x)$, RECT, TRAP, SIMP methods:
`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/solve-quadrature-numerically.pdf`

Wolfram Alpha at `http://www.wolframalpha.com/` can do the RECT rule and graphics with input string

```
integrate sin(x^2) using left endpoint method with interval width 0.2 from
x=0 to x=1
```