Chapter 2. Sample Problem 5.

$u' = u(u^2-4)$

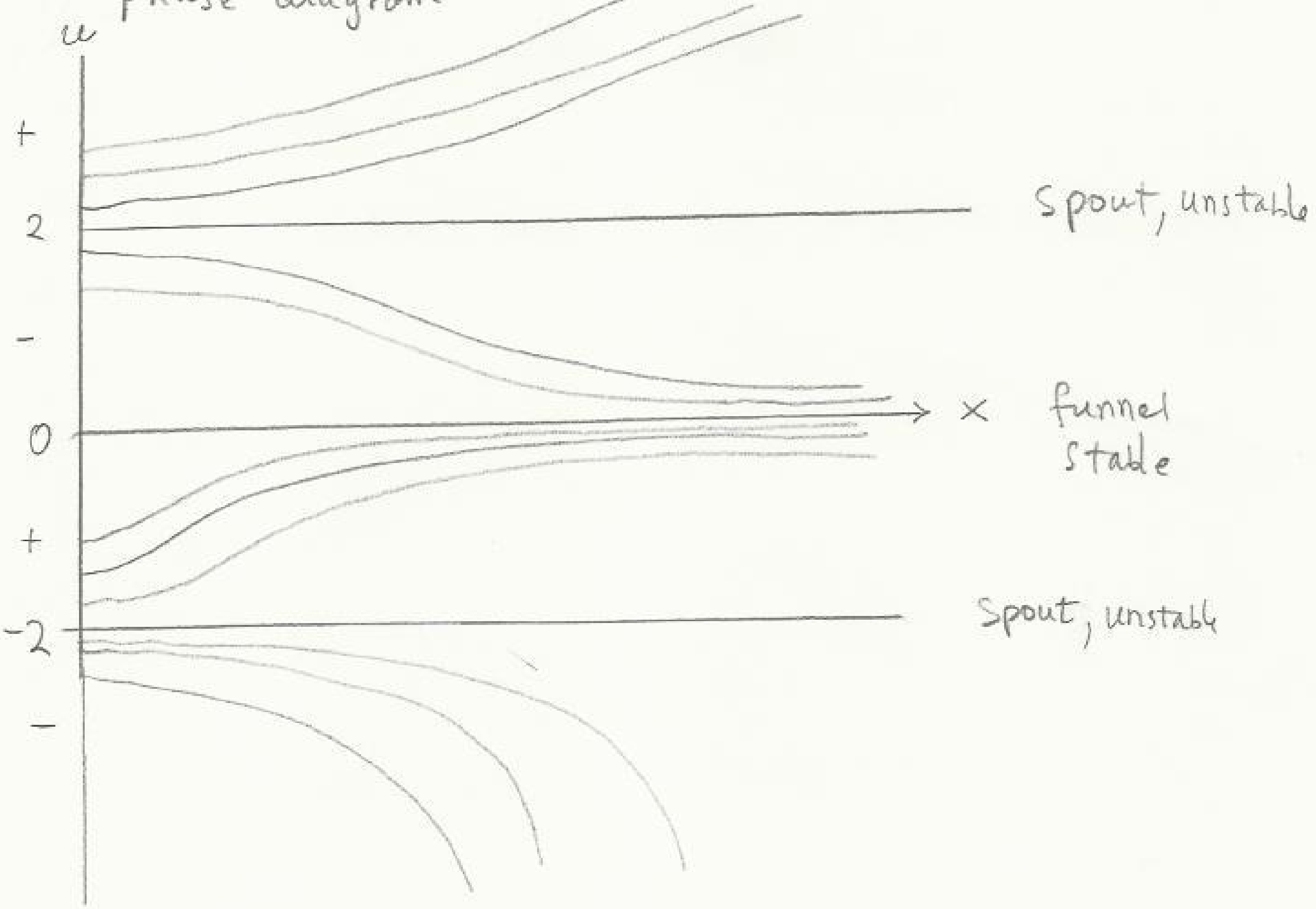
<u>phase line diagram</u>, $F(u) = u(u^2-4) = u(u-2)(u+2)$

Equilibria are roots of $F(u)=0$. Then $u = 0, 2, -2$

$F(-3) =$ Neg, $F(1) =$ Neg
$F(-1) =$ pos, $F(3) =$ pos
Example: Sign − on $0<u<2$
because $F(1) = -3$ (F is one-signed for $0<u<2$)

The phase line diagram was constructed from F. We move it to the vertical u-axis, below. Sign + means increasing, − means decreasing.

phase diagram



<u>Theorem</u> Between 2 adjacent equilibrium points, $F(u)$ is one-signed. Alternate: if $F(a)=F(b)=0$ and F has no roots in $a<u<b$, Then F is one-signed on $a<u<b$.

Chapter 2. Sample Problem 6.

$$\begin{cases} y' = \sin(x^2) \\ y(0) = 0 \end{cases}$$

Solve numerically using Rect, Trap, Simp

RECT The approx formula is $y(x_0+h) = y(x_0) + h\,F(x_0)$, because $\int_a^b F\,dx \cong F(a)(b-a)$ for $b-a$ small. Then the supplied table implies

$$y(0.6) = y(0.4) + \int_{0.4}^{0.6} F\,dx \cong 0.007997866838 + (0.2)\,F(0.4)$$
$$= 0.007997866838 + (0.2)\sin(0.4^2)$$
$$= 0.03986150816$$
$$y(0.8) = 0.03986150816 + 0.2\sin(0.6^2)$$
$$= 0.1103163548$$

TRAP The approx formula $\int_a^b F\,dx \cong (b-a)\frac{F(a)+F(b)}{2}$ implies that $y(x_0+h) = y(x_0) + \frac{h}{2}\left(F(x_0) + F(x_0+h)\right)$. The Table implies

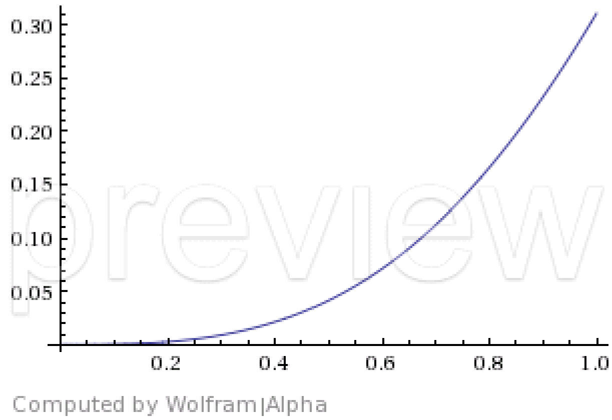
$$y(0.2) = y(0.0) + \frac{0.2}{2}\left(F(0) + F(0.2)\right)$$
$$= 0 + 0.1\left(\sin(0) + \sin(0.2^2)\right)$$
$$= 0.003998933419$$
$$y(0.8) = y(0.6) + 0.1\left(F(0.6) + F(0.8)\right)$$
$$= 0.07508893150 + 0.1\left(\sin(0.6^2) + \sin(0.8^2)\right)$$
$$= 0.1700358989$$

RK4 Use $\int_a^b F\,dx \cong \frac{(b-a)}{6}\left(F(a) + 4F\left(\frac{a+b}{2}\right) + F(b)\right)$ to get $y(x_0+h) = y(x_0) + \frac{h}{6}\left(F(x_0) + 4F(x_0+h/2) + F(x_0+h)\right)$. The Table implies

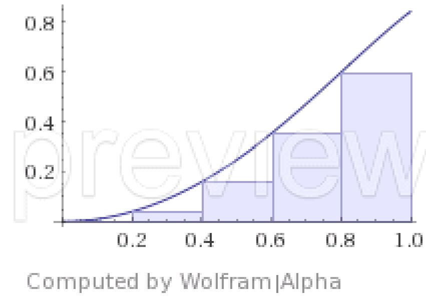
$$y(0.6) = y(0.4) + \frac{0.2}{6}\left(F(0.4) + 4F(0.5) + F(0.6)\right)$$
$$= 0.2129368017 + \frac{0.1}{3}\left(\sin(0.4^2) + 4\sin(0.5^2) + \sin(0.6^2)\right)$$
$$= 0.07133395608$$
$$y(1.0) = y(0.8) + \frac{0.1}{3}\left(\sin(0.8^2) + 4\sin(0.9^2) + \sin(1.0^2)\right)$$
$$= 0.3102602343$$

Solution Key

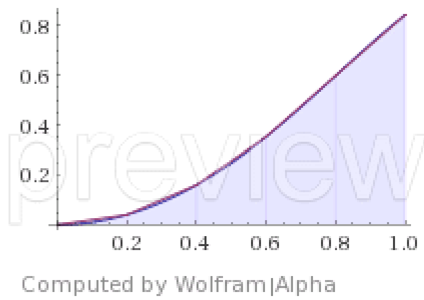
0.003998933 4	0.03986150 8	0.110316354	
		0.1700358989	
	0.0713339560 8		0.3102602343



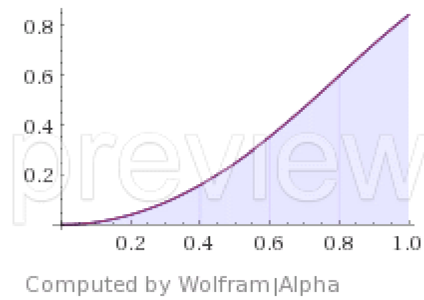
10-digit integral of sin(x^2)



RECT rule plot of y'=sin(x^2), h=0.2



TRAP rule plot of y'=sin(x^2), h=0.2



SIMP rule plot of y'=sin(x^2), h=0.2