Sample Quizz S2014 math 2250

problem 1

Answers: (1) $u = 68 + 117 e^{-ht}$

(2) $h = (-1/50) \ln\left(\frac{113}{117}\right) \cong 0.0006957$

(3) $t \cong 345.52$ seconds, about 6 min.

Details (1). Because u = degrees F and t = seconds, then the model is $\begin{cases} u' = -h(u-68), \\ u(0) = 185, \quad u(50) = 181. \end{cases}$

The DE is solved by superposition $u = u_h + u_p$. The equilibrium solution is $u_p = 68$. Then $u_h = \frac{c}{\text{integ. factor}} = c e^{-ht}$, using standard linear form $u' + hu = 68h$. condition $u(0) = 185$ is used on $u = u_h + u_p = ce^{-ht} + 68$ to evaluate $185 = ce^0 + 68$, Then $c = 185 - 68 = 117$.

Details (2). Start with answer (1) and use $u(50) = 181$. Then

$$181 = 68 + 117 e^{-ht} \qquad \text{when } t = 50$$
$$113 = 117 e^{-ht}$$
$$e^{-ht} = \frac{113}{117} \Rightarrow -ht = \ln\left(\frac{113}{117}\right)$$
$$\Rightarrow h = \frac{-1}{50} \ln\left(\frac{113}{117}\right)$$

Details (3). As in details (2),

$$160 = 68 + 117 e^{-ht}$$
$$e^{-ht} = \frac{160-68}{117}$$
$$-ht = \ln\left(\frac{92}{117}\right) \qquad \text{take log across eqtn.}$$
$$t = \left(\frac{-1}{h}\right) \ln\left(\frac{92}{117}\right)$$
$$t = 50 \frac{\ln(92/117)}{\ln(113/117)} \cong 345.52 \text{ seconds}$$

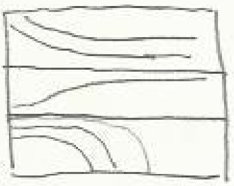
Sample Quiz 2 S2014 Math 2250

problem 2

Answers (1) $x' = 0.8x(1 - \frac{x}{780500})$, $x = 0, 780500$

(2) $x^2 - kx + \frac{HK}{r} = 0$, roots $= \frac{K}{2} \pm \frac{1}{2}\sqrt{K^2 - \frac{4HK}{r}}$

(3)

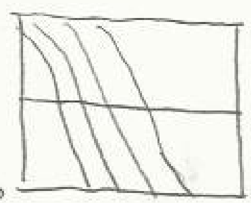


$a = \frac{K}{2} + \frac{1}{2}\sqrt{D}$

$b = \frac{K}{2} - \frac{1}{2}\sqrt{D}$

$D = K^2 - \frac{4HK}{r}$

Choose $H = \frac{Kr}{4} - 5000$



$a = b = \frac{Kr}{4}$ Not a sol

Choose $H = \frac{Kr}{4} + 5000$
All sols decrease to zero (and beyond), meaning extinction

Details (1). $x' = rx(1 - \frac{x}{K})$, Substitute $r = 0.8$, $K = 780500$

Details (2). $x' = rx(1 - \frac{x}{K}) - H = rx - \frac{r}{K}x^2 - H$

$x' = -\frac{r}{K}(x^2 - Kx + \frac{HK}{r})$

Apply quadratic formula to $x^2 - Kx + \frac{HK}{r} = 0$ to find roots, reported in both (2), (3) above.

Details (3). A double real root is when the discriminant $D = K^2 - \frac{4HK}{r} = 0$, requiring $H = \frac{Kr}{4}$. For $H < \frac{Kr}{4}$, there are 2 real roots a, b as given in the answer. For $H > \frac{Kr}{4}$ there are no real roots, therefore $x' < 0$ and x decreases to zero (extinction) and beyond.

The larger root $a = \frac{K}{2} + \frac{1}{2}\sqrt{D}$ is a stable funnel.
The smaller root $b = \frac{K}{2} - \frac{1}{2}\sqrt{D}$ is an unstable spout.
When $D = 0$, then $a = b = K/2$ is a node. When a, b are real, extinction for $x < b$, carrying capacity $= a$, sustainable harvest for $x > b$.