Chapter 1. Sample Problem 1

An answer check for the differential equation and initial condition

$$\frac{dy}{dx} = -y(x) + 23, \quad y(0) = 5 \tag{1}$$

requires substitution of the candidate solution $y(x) = 23 - 18\,e^{-x}$ into the left side (LHS) and right side (RHS), then compare the expressions for equality for all symbols. The process of testing LHS = RHS applies to both the differential equation and the initial condition, making the answer check have **two** presentation panels. Complete the following:

1. Show the two panels in an answer check for initial value problem (1).
2. Relate (1) to a Newton cooling model for warming a 5 C apple to room temperature 23 C.

References. Edwards-Penney sections 1.1, 1.4, 1.5. Newton cooling in Serway and Vuille, *College Physics 9/E*, Brooks-Cole (2011), ISBN-10: 0840062060.
Newton cooling differential equation $\frac{du}{dt} = -h(u(t) - u_1)$, slide:
`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/2250ThreeExamples.pdf`
Slide on answer checks:
`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/FTC-Method-of-Quadrature.pdf`

Chapter 1. Sample Problem 2

A 2-ft high institutional coffee maker serves coffee from an orifice 5 inches above the base of the cylindrical tank. The tank drains according to the Torricelli model

$$\frac{dy}{dx} = -0.02\sqrt{|y(x)|}, \quad y(0) = y_0. \tag{2}$$

Symbol $y(x) \geq 0$ is the tank coffee height in feet above the orifice at time x seconds, while $y_0 \geq 0$ is the coffee height at time $x = 0$.

Establish these facts about the physical problem.

1. If $y_0 = 0$, then $y(x)$ is not determined by the model. A physical explanation is expected, based on possible past tank levels. Numerical solutions are therefore technological nonsense.
2. If $y_0 > 0$, then the solution $y(x)$ is uniquely determined and computable by numerical software. Justify using Picard's existence-uniqueness theorem.
3. Solve equation (2) using separation of variables when y_0 is 19 inches, then numerically find the drain time (about 125 seconds).

References. Edwards-Penney, Picard's theorem 1 section 1.3 and Torricelli's Law section 1.4. Tank draining `Mathematica` demo at Wolfram Research:

Carl Schaschke, *Fluid Mechanics: Worked Examples for Engineers*, The Institution of Chemical Engineers (2005), ISBN-10: 0852954980, Chapter 6.
Slide on Picard and Peano Theorems:
`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/Picard+DirectionFields.pdf`