Theorem 1 (Peano)
Let (x_0, y_0) be the center of a box

$$B = \{(x, y) : |x - x_0| \le H, |y - y_0| \le K\}$$

and assume $f(x, y)$ is continuous on B. Then there is a small $h > 0$ and a function $y(x)$ continuously differentiable on $|x - x_0| < h$ such that $(x, y(x))$ remains in B for $|x - x_0| < h$ and $y(x)$ is one solution (many more might exist) of the initial value problem

$$y' = f(x, y), \quad y(x_0) = y_0.$$

Definition 1 (Picard Iteration)
Define the constant function $y_0(x) = y_0$ and then define by iteration

$$y_{n+1}(x) = y_0 + \int_{x_0}^{x} f(t, y_n(t))dt.$$

The sequence $y_0(x)$, $y_1(x)$, ... is called the **sequence of Picard iterates** for $y' = f(x, y)$, $y(x_0) = y_0$.

Theorem 2 (Picard-Lindelöf)
Let (x_0, y_0) be the center of a box

$$B = \{(x, y) : |x - x_0| \leq H, |y - y_0| \leq K\}$$

and assume $f(x, y)$ and $f_y(x, y)$ are continuous on B. Then there is a small $h > 0$ and a *unique* function $y(x)$ continuously differentiable on $|x - x_0| < h$ such that $(x, y(x))$ remains in B for $|x - x_0| < h$ and $y(x)$ solves

$$y' = f(x, y), \quad y(x_0) = y_0.$$

The equation

$$\lim_{n \to \infty} y_n(x) = y(x)$$

is satisfied for $|x - x_0| < h$ by the Picard iterates y_n.