Title: Using Linear Algebra to Rank the Men’s Short Track Speedskating World Cup Relay Teams in the 2017-2018 Season

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Abstract

The goal of this paper was to create a novel ranking of the men’s short track speedskating World Cup relay teams based on the first four World Cups of the 2017-2018 season. Crucially, these are the four world-level competitions preceding the Olympic Games, and thus rank is vital for team goal setting, seeding for future competitions, and predicting future outcomes, etc. A novel ranking—predicated on margin of head-to-head victory in seconds—was achieved and will be discussed in the following pages.

Methods

A crude version of the Google page rank algorithm was used to rank the top ten men’s relay teams in the world. Ten teams were chosen, as only the top eight teams in the world can qualify to skate in the Olympic Games. Thus, expanding the rank beyond ten teams is unnecessary, especially given the fact that competitiveness drastically falls off after the top ten or so teams have been considered.

To start, data was gathered and organized in an excel spreadsheet. The spreadsheet contained every relay race of the first four world cups of the 2017-2018 season, and included the order of finish and the associated times of each country. Next, time differential behind the winner of each race was calculated for each team. This time differential is the vital statistic in the novel ranking I attempted to create. A 10x10 matrix named A was created to compare the results of the ten teams. The entries in A, A_{ij}, were defined as the winning margin (in seconds) of team i over team j. The winning margin was an aggregate time (in seconds) of all meetings between team i and team j in which team i won the race. If team i never beat team j, a zero was entered into the matrix. Note that the row index is as follows: CAN, KOR, USA, CHN, NED, JPN, RUS, HUN, KAZ, BEL. The column index is identical. The abbreviations used are those used by the ISU (International Skating Union) and can be accessed using the links provided in the works consulted section.

Next, the column vectors of A were extracted and subsequently normalized so that their components summed to one. The resulting vectors S1, S2,…, S10 were then augmented back together to form the normalized matrix of A, called S. Once this had been accomplished, the eigenvalues of S were calculated. The dominant eigenvalue is the one with the greatest magnitude; this was selected from the given eigenvalues. The dominant eigenvalue was then stripped of any imaginary component it had acquired in the calculation process (likely due to rounding error, especially with such small numbers as initial entries). Next, the dominant eigenvector was calculated using the associated dominant eigenvalue. Finally, this dominant eigenvector was renamed the ranking vector. Absolute value was applied to the raking vector, as magnitude is the important component of each entry in this inquiry. To elucidate the new ranking, the entries in the ranking vector were placed in ascending order, as were the corresponding countries. When teams had the same ranking vector entry (this only occurred with entry = 0), they were arbitrarily ordered below any team with a nonzero ranking vector entry.

Calculations

Most calculations were performed in the mathematics program Maple, although preliminary data collection and number crunching was done in Microsoft Excel.

Heat	Country	Place	Time (sec)	Place Differential	Time Differential
1	RUS	1	414.955	0	0
1	KAZ	2	415.838	1	0.883
1	LAT	3	417.075	2	2.12
1	TUR	4	427.73	3	12.775
2	HUN	1	411.814	0	0
2	USA	2	412.538	1	0.724
2	GBR	3	420.094	2	8.28
2	UKR	4	436.683	3	24.869
3	KOR	1	411.348	0	0
3	CAN	2	412.199	1	0.851
3	AUS	3	420.501	2	9.153
3	BLR	4	424.103	3	12.755
4	NED	1	407.353	0	0
4	JPN	2	407.428	1	0.075
4	ITA	3	407.718	2	0.365
4	AUT	4	DNF	3	#VALUE!
5	CHN	1	414.072	0	0
5	FRA	2	414.81	1	0.738
5	BEL	3	416.302	2	2.23
5	POL	4	423.097	3	9.025

Quarter	Country	Place	Time (sec)	Place Differential	Time Differential
1	NED	1	404.961	0	0
1	CAN	2	405.162	1	0.201
1	JPN	3	405.456	2	0.495
1	BEL	4	427.289	3	22.328
2	CHN	1	430.738	0	0
2	HUN	2	431.163	1	0.425
2	KAZ	3	431.779	2	1.041
2	FRA	4	PEN	3	#VALUE!
3	KOR	1	413.348	0	0
3	USA	2	414.988	1	1.64
3	RUS	3	415.055	2	1.707
3	ITA	4	415.264	3	1.916

Semi	Country	Place	Time (sec)	Place Differential	Time Differential
1	KOR	1	414.363	0	0
1	CHN	2	414.455	1	0.092
1	RUS	3	414.774	2	0.411
1	USA	4	415.18	3	0.817
2	CAN	1	414.285	0	0
2	JPN	2	414.728	1	0.443
2	KAZ	3	414.74	2	0.455
2	HUN	4	431.112	3	16.827
2	NED	5	PEN		#VALUE!

Final	Country	Place	Time (sec)	Place Differential	Time Differential
1	CAN	1	406.191	0	0
1	CHN	2	406.196	1	0.005
1	JPN	3	407.825	2	1.634
1	KOR	4	409.201	3	3.01
2	HUN	1	415.913	0	0
2	USA	2	416.057	1	0.144
2	KAZ	3	416.251	2	0.338
2	RUS	4	418.234	3	2.321

Heat	Country	Place	Time (sec)	Place Differential	Time Differential
1	RUS	1	419.042	0	0
1	KAZ	2	419.417	1	0.375
1	POL	3	423.429	2	4.387
1	BLR	4	433.7	3	14.658
2	CAN	1	405.019	0	0
2	FRA	2	405.232	1	0.213
2	ITA	3	418.519	2	13.5
2	AUT	4	PEN	3	#VALUE!
3	KOR	1	409.406	0	0
3	CHN	2	409.434	1	0.028
3	LAT	3	416.154	2	6.748
3	TUR	4	PEN	3	#VALUE!
4	NED	1	406.716	0	0
4	JPN	2	408.956	1	2.24
4	BEL	3	412.239	2	5.523
4	UKR	4	DNF	3	#VALUE!
5	HUN	1	410.325	0	0
5	USA	2	411.633	1	1.308
5	GBR	3	416.132	2	5.807
5	AUS	4	444.113	3	33.788

Quarter	Country	Place	Time (sec)	Place Differential	Time Differential
1	CHN	1	404.901	0	0
1	NED	2	405.406	1	0.505
1	RUS	3	405.897	2	0.996
1	BEL	4	422.262	3	17.361
1	ITA	5	PEN	4	#VALUE!
2	KOR	1	406.03	0	0
2	HUN	2	406.052	1	0.022
2	USA	3	406.149	2	0.119
2	KAZ	4	406.374	3	0.344
3	CAN	1	406.197	0	0
3	JPN	2	406.633	1	0.436
3	LAT	3	420.515	2	14.318
3	FRA	4	PEN	3	#VALUE!
3	GRB	5	PEN	4	#VALUE!

Semi	Country	Place	Time (sec)	Place Differential	Time Differential
1	CAN	1	403.81	0	0
1	NED	2	404.275	1	0.465
1	USA	3	404.349	2	0.539
1	BEL	4	411.847	3	8.037
1	HUN	5	414.041	4	10.231
2	JPN	1	419.257	0	0
2	RUS	2	419.502	1	0.245
2	CHN	3	419.862	2	0.605
2	KOR	4	PEN	3	#VALUE!

Final	Country	Place	Time (sec)	Place Differential	Time Differential
1	CAN	1	402.478	0	0
1	NED	2	402.554	1	0.076
1	CHN	3	402.746	2	0.268
1	JPN	4	403.008	3	0.53
1	RUS	5	405.44	4	2.962
2	USA	1	413.984	0	0
2	BEL	2	416.553	1	2.569

Quarter	Country	Place	Time (sec)	Place Differential	Time Differential		Semi	Country	Place	Time (sec)	Place Differential	Time Differential		Final	Country	Place	Time (sec)	Place Differential	Time Differential
1	CHN	1	407.286	0	0		1	KOR	1	401.201	0	0		1	USA	1	389.052	0	0
1	ITA	2	407.75	1	0.464		1	CHN	2	401.202	1	0.001		1	KOR	2	389.076	1	0.024
1	GBR	3	413.376	2	6.09		1	RUS	3	401.452	2	0.251		1	CAN	3	399.805	2	10.753
1	JPN	4	PEN	3	#VALUE!		1	ITA	4	401.768	3	0.567		1	CHN	4	PEN	3	#VALUE!
2	KOR	1	412.44	0	0		2	CAN	1	403.094	0	0		2	RUS	1	399.76	0	0
2	RUS	2	413.994	1	1.554		2	USA	2	403.198	1	0.104		2	HUN	2	400.367	1	0.607
2	TUR	3	426.229	2	13.789		2	HUN	3	421.069	2	17.975		2	ITA	3	411.654	2	11.894
2	BEL	4	PEN	3	#VALUE!		2	FRA	4	PEN	3	#VALUE!							
3	HUN	1	403.882	0	0														
3	FRA	2	404.074	1	0.192														
3	NED	3	404.672	2	0.79														
3	AUS	4	PEN	3	#VALUE!														
4	CAN	1	416.002	0	0														
4	USA	2	420.054	1	4.052														
4	KAZ	3	PEN	2	#VALUE!														

Quarter	Country	Place	Time (sec)	Place Differential	Time Differential
1	NED	1	404.082	0	0
1	JPN	2	404.127	1	0.045
1	FRA	3	412.706	2	8.624
1	AUS	4	415.242	3	11.16
2	USA	1	402.414	0	0
2	KOR	2	406.995	1	4.581
2	GBR	3	419.042	2	16.628
2	ITA	4	PEN	3	#VALUE!
3	HUN	1	414.208	0	0
3	KAZ	2	415.602	1	1.394
3	CAN	3	421.44	2	7.232
3	AUT	4	423.848	3	9.64
4	RUS	1	419.893	0	0
4	BEL	2	421.964	1	2.071
4	CHN	3	425.366	2	5.473
4	TUR	4	435.899	3	16.006

Semi	Country	Place	Time (sec)	Place Differential	Time Differential
1	RUS	1	415.42	0	0
1	USA	2	416.491	1	1.071
1	BEL	3	418.745	2	3.325
1	JPN	4	421.375	3	5.955
2	NED	1	407.358	0	0
2	KOR	2	407.472	1	0.114
2	KAZ	3	434.611	2	27.253
2	HUN	4	PEN	3	#VALUE!

Final	Country	Place	Time (sec)	Place Differential	Time Differential
1	KOR	1	407.365	0	0
1	NED	2	407.501	1	0.136
1	USA	3	407.894	2	0.529
1	RUS	4	408.535	3	1.17
1	KAZ	5	409.999	4	2.634
2	JPN	1	419.259	0	0
2	BEL	2	423.106	1	3.847

Winning Margin When Placing First											
		CAN	KOR	USA	CHN	NED	JPN	RUS	HUN	KAZ	BEL
	CAN	0	3.01	4.695	0.273	0.541	3.043	2.962	45.033	0.455	8.037
		CAN	KOR	USA	CHN	NED	JPN	RUS	HUN	KAZ	BEL
	KOR	0.851	0	3.105	0.121	0.136	0	5.093	0.022	2.987	0
		CAN	KOR	USA	CHN	NED	JPN	RUS	HUN	KAZ	BEL
	USA	10.753	4.605	0	0	0	0	0	0	0	2.569
		CAN	KOR	USA	CHN	NED	JPN	RUS	HUN	KAZ	BEL
	CHN	2.23	0	0	0	0.505	0	0	0.425	1.041	0
		CAN	KOR	USA	CHN	NED	JPN	RUS	HUN	KAZ	BEL
	NED	0.201	0.114	0	0	0	2.855	0	0	27.253	27.851
		CAN	KOR	USA	CHN	NED	JPN	RUS	HUN	KAZ	BEL
	JPN	0	0	0	0.605	0	0	0.245	0	0	0
		CAN	KOR	USA	CHN	NED	JPN	RUS	HUN	KAZ	BEL
	RUS	0	0	1.071	5.473	0	5.955	0	0.607	1.258	5.396
		CAN	KOR	USA	CHN	NED	JPN	RUS	HUN	KAZ	BEL
	HUN	7.232	0	2.176	0	0.79	0	2.321	0	1.732	0
		CAN	KOR	USA	CHN	NED	JPN	RUS	HUN	KAZ	BEL
	KAZ	0	0	0	0	0	0	0	0	0	0
		CAN	KOR	USA	CHN	NED	JPN	RUS	HUN	KAZ	BEL
	BEL	0	0	0	0	0	0	0	0	0	0

```
> # Tyler Kroll
> # Semester Project Calculations
>
> A:=<<0,.851,10.753,2.23,.201,0,0,7.232,0,0>|<3.01,0,4.605,0,.114,0,
  0,0,0,0>|<4.695,3.105,0,0,0,0,1.071,2.176,0,0>|<.273,.121,0,0,0,
  .605,5.473,0,0,0>|<.541,.136,0,.505,0,0,0,.79,0,0>|<3.043,0,0,0,
  2.855,0,5.955,0,0,0>|<2.962,5.093,0,0,0,.245,0,2.321,0,0>|<45.033,
  .022,0,.425,0,0,.607,0,0,0>|<.455,2.987,0,1.041,27.253,0,1.258,
  1.732,0,0>|<8.037,0,2.569,0,27.851,0,5.396,0,0,0>>;
```

$$A := \begin{bmatrix} 0 & 3.01 & 4.695 & 0.273 & 0.541 & 3.043 & 2.962 & 45.033 & 0.455 & 8.037 \\ 0.851 & 0 & 3.105 & 0.121 & 0.136 & 0 & 5.093 & 0.022 & 2.987 & 0 \\ 10.753 & 4.605 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.569 \\ 2.23 & 0 & 0 & 0 & 0.505 & 0 & 0 & 0.425 & 1.041 & 0 \\ 0.201 & 0.114 & 0 & 0 & 0 & 2.855 & 0 & 0 & 27.253 & 27.851 \\ 0 & 0 & 0 & 0.605 & 0 & 0 & 0.245 & 0 & 0 & 0 \\ 0 & 0 & 1.071 & 5.473 & 0 & 5.955 & 0 & 0.607 & 1.258 & 5.396 \\ 7.232 & 0 & 2.176 & 0 & 0.79 & 0 & 2.321 & 0 & 1.732 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (1)$$

```
> A1:=A[1..10,1];
```

$$A1 := \begin{bmatrix} 0 \\ 0.851 \\ 10.753 \\ 2.23 \\ 0.201 \\ 0 \\ 0 \\ 7.232 \\ 0 \\ 0 \end{bmatrix} \quad (2)$$

```
> A2:=A[1..10,2];
```

(3)

$$A2 := \begin{bmatrix} 3.01 \\ 0 \\ 4.605 \\ 0 \\ 0.114 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

```
> A3:=A[1..10,3];
```

$$A3 := \begin{bmatrix} 4.695 \\ 3.105 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1.071 \\ 2.176 \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

```
> A4:=A[1..10,4];
```

$$A4 := \begin{bmatrix} 0.273 \\ 0.121 \\ 0 \\ 0 \\ 0 \\ 0.605 \\ 5.473 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

```
> A5:=A[1..10,5];
```

$$A5 := \begin{bmatrix} 0.541 \\ 0.136 \\ 0 \\ 0.505 \\ 0 \\ 0 \\ 0 \\ 0.79 \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

```
> A6:=A[1..10,6];
```

$$A6 := \begin{bmatrix} 3.043 \\ 0 \\ 0 \\ 0 \\ 2.855 \\ 0 \\ 5.955 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

```
> A7:=A[1..10,7];
```

$$A7 := \begin{bmatrix} 2.962 \\ 5.093 \\ 0 \\ 0 \\ 0 \\ 0.245 \\ 0 \\ 2.321 \\ 0 \\ 0 \end{bmatrix} \quad (8)$$

```
> A8:=A[1..10,8];
```

$$A8 := \begin{bmatrix} 45.033 \\ 0.022 \\ 0 \\ 0.425 \\ 0 \\ 0 \\ 0.607 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

```
> A9:=A[1..10,9];
```

$$A9 := \begin{bmatrix} 0.455 \\ 2.987 \\ 0 \\ 1.041 \\ 27.253 \\ 0 \\ 1.258 \\ 1.732 \\ 0 \\ 0 \end{bmatrix} \quad (10)$$

```
> A10:=A[1..10,10];
```

$$A10 := \begin{bmatrix} 8.037 \\ 0 \\ 2.569 \\ 0 \\ 27.851 \\ 0 \\ 5.396 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

```
> with(LinearAlgebra);
```

[*&x*, *Add*, *Adjoint*, *BackwardSubstitute*, *BandMatrix*, *Basis*, *BezoutMatrix*, *BidiagonalForm*, *BilinearForm*, *CARE*, *CharacteristicMatrix*, *CharacteristicPolynomial*, *Column*, **(12)** *ColumnDimension*, *ColumnOperation*, *ColumnSpace*, *CompanionMatrix*, *CompressedSparseForm*, *ConditionNumber*, *ConstantMatrix*, *ConstantVector*, *Copy*, *CreatePermutation*, *CrossProduct*, *DARE*, *DeleteColumn*, *DeleteRow*, *Determinant*, *Diagonal*, *DiagonalMatrix*, *Dimension*, *Dimensions*, *DotProduct*, *EigenConditionNumbers*, *Eigenvalues*, *Eigenvectors*, *Equal*, *ForwardSubstitute*, *FrobeniusForm*, *FromCompressedSparseForm*, *FromSplitForm*, *GaussianElimination*, *GenerateEquations*, *GenerateMatrix*, *Generic*, *GetResultDataType*, *GetResultShape*, *GivensRotationMatrix*, *GramSchmidt*, *HankelMatrix*, *HermiteForm*, *HermitianTranspose*, *HessenbergForm*, *HilbertMatrix*, *HouseholderMatrix*, *IdentityMatrix*, *IntersectionBasis*, *IsDefinite*, *IsOrthogonal*, *IsSimilar*, *IsUnitary*, *JordanBlockMatrix*, *JordanForm*, *KroneckerProduct*, *LA_Main*, *LUDecomposition*, *LeastSquares*, *LinearSolve*, *LyapunovSolve*, *Map*, *Map2*, *MatrixAdd*, *MatrixExponential*, *MatrixFunction*, *MatrixInverse*, *MatrixMatrixMultiply*, *MatrixNorm*, *MatrixPower*, *MatrixScalarMultiply*, *MatrixVectorMultiply*, *MinimalPolynomial*, *Minor*, *Modular*, *Multiply*, *NoUserValue*, *Norm*, *Normalize*, *NullSpace*, *OuterProductMatrix*, *Permanent*, *Pivot*, *PopovForm*, *ProjectionMatrix*, *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *RationalCanonicalForm*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, *SmithForm*, *SplitForm*, *StronglyConnectedBlocks*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *SylvesterSolve*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

```
> SA1:=add(A1(n),n=1..10);
```

$$SA1 := 21.267 \qquad (13)$$

```
> SA2:=add(A2(n),n=1..10);
```

$$SA2 := 7.729 \qquad (14)$$

```
> SA3:=add(A3(n),n=1..10);
```

$$SA3 := 11.047 \qquad (15)$$

```
> SA4:=add(A4(n),n=1..10);
```

$$SA4 := 6.472 \qquad (16)$$

```
> SA5:=add(A5(n),n=1..10);
```

$$SA5 := 1.972 \qquad (17)$$

```
> SA6:=add(A6(n),n=1..10);
```

$$SA6 := 11.853 \qquad (18)$$

```
> SA7:=add(A7(n),n=1..10);
```

$$SA7 := 10.621 \qquad (19)$$

```
> SA8:=add(A8(n),n=1..10);
```

$$SA8 := 46.087 \qquad (20)$$

```
> SA9:=add(A9(n),n=1..10);
```

$$SA9 := 34.726 \qquad (21)$$

```
> SA10:=add(A10(n),n=1..10);
```

$$SA10 := 43.853 \quad (22)$$

```
> S1:=A1/SA1;
```

$$S1 := \begin{bmatrix} 0. \\ 0.0400150467825600 \\ 0.505619034139680 \\ 0.104857290628800 \\ 0.00945126251856000 \\ 0. \\ 0. \\ 0.340057365841920 \\ 0. \\ 0. \end{bmatrix} \quad (23)$$

```
> S2:=A2/SA2;
```

$$S2 := \begin{bmatrix} 0.389442359838000 \\ 0. \\ 0.595807995699000 \\ 0. \\ 0.0147496441932000 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{bmatrix} \quad (24)$$

```
> S3:=A3/SA3;
```

$$S3 := \begin{bmatrix} 0.425002263056250 \\ 0.281071784193750 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0.0969493980262500 \\ 0.196976554720000 \\ 0. \\ 0. \end{bmatrix} \quad (25)$$

```
> S4:=A4/SA4;
```

$$S4 := \begin{bmatrix} 0.0421817058117000 \\ 0.0186959208909000 \\ 0. \\ 0. \\ 0. \\ 0.0934796044545000 \\ 0.845642768891700 \\ 0. \\ 0. \\ 0. \end{bmatrix} \quad (26)$$

```
> S5:=A5/SA5;
```

$$S5 := \begin{bmatrix} 0.274340770801500 \\ 0.0689655172440000 \\ 0. \\ 0.256085192707500 \\ 0. \\ 0. \\ 0. \\ 0.400608519285000 \\ 0. \\ 0. \end{bmatrix} \quad (27)$$

```
> S6:=A6/SA6;
```

$$S6 := \begin{bmatrix} 0.256728254439280 \\ 0. \\ 0. \\ 0. \\ 0.240867290970800 \\ 0. \\ 0.502404454546800 \\ 0. \\ 0. \\ 0. \end{bmatrix} \quad (28)$$

```
> S7:=A7/SA7;
```

$$S7 := \begin{bmatrix} 0.278881461258660 \\ 0.479521702292490 \\ 0. \\ 0. \\ 0. \\ 0.0230675077678500 \\ 0. \\ 0.218529328690530 \\ 0. \\ 0. \end{bmatrix} \quad (29)$$

```
> S8:=A8/SA8;
```

$$S8 := \begin{bmatrix} 0.977130210360420 \\ 0.000477358040280000 \\ 0. \\ 0.00922168941450000 \\ 0. \\ 0. \\ 0.0131707422931800 \\ 0. \\ 0. \\ 0. \end{bmatrix} \quad (30)$$

```
> S9:=A9/SA9;
```

$$S9 := \begin{bmatrix} 0.0131025744395000 \\ 0.0860162414303000 \\ 0. \\ 0.0299775384429000 \\ 0.784801013625700 \\ 0. \\ 0.0362264585602000 \\ 0.0498761734708000 \\ 0. \\ 0. \end{bmatrix} \quad (31)$$

```
> S10:=A10/SA10;
```

$$S10 := \begin{bmatrix} 0.183271383909000 \\ 0. \\ 0.0585820810330000 \\ 0. \\ 0.635099080907000 \\ 0. \\ 0.123047453972000 \\ 0. \\ 0. \\ 0. \end{bmatrix} \quad \textbf{(32)}$$

```
> interface(rtablesize=infinity);
```

10 **(33)**

```
> S:=<S1|S2|S3|S4|S5|S6|S7|S8|S9|S10>;
```

S := [[0., 0.389442359838000, 0.425002263056250, 0.0421817058117000, 0.274340770801500, 0.256728254439280, 0.278881461258660, 0.977130210360420, 0.0131025744395000, 0.183271383909000], **(34)**
[0.0400150467825600, 0., 0.281071784193750, 0.0186959208909000, 0.0689655172440000, 0., 0.479521702292490, 0.000477358040280000, 0.0860162414303000, 0.],
[0.505619034139680, 0.595807995699000, 0., 0., 0., 0., 0., 0., 0., 0.0585820810330000],
[0.104857290628800, 0., 0., 0., 0.256085192707500, 0., 0., 0.00922168941450000, 0.0299775384429000, 0.],
[0.00945126251856000, 0.0147496441932000, 0., 0., 0., 0.240867290970800, 0., 0., 0.784801013625700, 0.635099080907000],
[0., 0., 0., 0.0934796044545000, 0., 0., 0.0230675077678500, 0., 0., 0.],
[0., 0., 0.0969493980262500, 0.845642768891700, 0., 0.502404454546800, 0., 0.0131707422931800, 0.0362264585602000, 0.123047453972000],
[0.340057365841920, 0., 0.196976554720000, 0., 0.400608519285000, 0., 0.218529328690530, 0., 0.0498761734708000, 0.],
[0., 0., 0., 0., 0., 0., 0., 0., 0., 0.],
[0., 0., 0., 0., 0., 0., 0., 0., 0., 0.]]

```
> EigenvaluesofS:=Eigenvalues(S);
```

$$EigenvaluesofS := \begin{bmatrix} 0.999999999960023 + 0.\,I \\ -0.624222094487519 + 0.\,I \\ -0.304277892144507 + 0.135316482119093\,I \\ -0.304277892144507 - 0.135316482119093\,I \\ 0.139079857419490 + 0.0898737605322084\,I \\ 0.139079857419490 - 0.0898737605322084\,I \\ -0.0226909180112372 + 0.167819557132057\,I \\ -0.0226909180112372 - 0.167819557132057\,I \\ 0.\,I \\ 0.\,I \end{bmatrix} \quad (35)$$

```
> DominantEigenvalue:=EigenvaluesofS[1];
```

$$DominantEigenvalue := 0.999999999960023 + 0.\,I \quad (36)$$

```
> RealDominantEigenvalue:=evalc(Re(DominantEigenvalue));
```

$$RealDominantEigenvalue := 0.999999999960023 \quad (37)$$

```
> ID:=IdentityMatrix(10,10);
```

$$ID := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (38)$$

```
> lambda:=RealDominantEigenvalue.ID;
```

λ := [[0.999999999960023, 0., 0., 0., 0., 0., 0., 0., 0., 0.], **(39)**
[0., 0.999999999960023, 0., 0., 0., 0., 0., 0., 0., 0.],
[0., 0., 0.999999999960023, 0., 0., 0., 0., 0., 0., 0.],
[0., 0., 0., 0.999999999960023, 0., 0., 0., 0., 0., 0.],
[0., 0., 0., 0., 0.999999999960023, 0., 0., 0., 0., 0.],
[0., 0., 0., 0., 0., 0.999999999960023, 0., 0., 0., 0.],
[0., 0., 0., 0., 0., 0., 0.999999999960023, 0., 0., 0.],
[0., 0., 0., 0., 0., 0., 0., 0.999999999960023, 0., 0.],
[0., 0., 0., 0., 0., 0., 0., 0., 0.999999999960023, 0.],
[0., 0., 0., 0., 0., 0., 0., 0., 0., 0.999999999960023]]

```
> EV:=S-lambda;
```

$EV :=$ [[−0.999999999960023, 0.389442359838000, 0.425002263056250, 0.0421817058117000, 0.274340770801500, 0.256728254439280, 0.278881461258660, 0.977130210360420, 0.0131025744395000, 0.183271383909000], [0.0400150467825600, −0.999999999960023, 0.281071784193750, 0.0186959208909000, 0.0689655172440000, 0., 0.479521702292490, 0.000477358040280000, 0.0860162414303000, 0.], [0.505619034139680, 0.595807995699000, −0.999999999960023, 0., 0., 0., 0., 0., 0., 0.0585820810330000], [0.104857290628800, 0., 0., −0.999999999960023, 0.256085192707500, 0., 0., 0.00922168941450000, 0.0299775384429000, 0.], [0.00945126251856000, 0.0147496441932000, 0., 0., −0.999999999960023, 0.240867290970800, 0., 0., 0.784801013625700, 0.635099080907000], [0., 0., 0., 0.0934796044545000, 0., −0.999999999960023, 0.0230675077678500, 0., 0., 0.], [0., 0., 0.0969493980262500, 0.845642768891700, 0., 0.502404454546800, −0.999999999960023, 0.0131707422931800, 0.0362264585602000, 0.123047453972000], [0.340057365841920, 0., 0.196976554720000, 0., 0.400608519285000, 0., 0.218529328690530, −0.999999999960023, 0.0498761734708000, 0.], [0., 0., 0., 0., 0., 0., 0., 0., −0.999999999960023, 0.], [0., 0., 0., 0., 0., 0., 0., 0., 0., −0.999999999960023]] **(40)**

```
> DominantEigenvector:=EV[1..10,1];
```

$$DominantEigenvector := \begin{bmatrix} -0.999999999960023 \\ 0.0400150467825600 \\ 0.505619034139680 \\ 0.104857290628800 \\ 0.00945126251856000 \\ 0. \\ 0. \\ 0.340057365841920 \\ 0. \\ 0. \end{bmatrix} \quad \textbf{(41)}$$

```
> RankingVector:=abs(DominantEigenvector);
```

$$
RankingVector := \begin{bmatrix} 0.999999999960023 \\ 0.0400150467825600 \\ 0.505619034139680 \\ 0.104857290628800 \\ 0.00945126251856000 \\ 0. \\ 0. \\ 0.340057365841920 \\ 0. \\ 0. \end{bmatrix} \quad \textbf{(42)}
$$

Results

ISU Ranking

1 CAN
2 KOR
3 USA
4 CHN
5 NED
6 JPN
7 RUS
8 HUN
9 KAZ
10 BEL

New Ranking

1 CAN
2 USA
3 HUN
4 CHN
5 KOR
6 NED
7 JPN
8 RUS
9 KAZ
10 BEL

Discussion

Generally, the traditional ISU ranking and the novel ranking agree with each other. Four of the top five teams are the same, and in a roughly similar order. Notably, Korea (KOR) fell down the ranking from second to fifth. This reflects the fact that, when going head-to-head with the USA, China (CHN), and Hungary (HUN), Korea has trouble. These results suggest that some aspect of teams USA, China, and Hungary pose a greater threat to team Korea than other countries. What characteristic account for that difference is purely speculative and will not be discussed here.

The most significant difference between the ISU's ranking and the new ranking is Hungary's jump from eighth place in the ISU ranking to third place in the new ranking. This suggest that Hungary, though inconsistent, beats other high-ranking teams by significant margins—but only when they win. In other words, when the Hungarian team brings its "A-game," they are a formidable force. However, they don't always skate their best, and thus their ISU ranking suffers. This analysis and interpretation of the data has been supported by subsequent real-world events: the Hungarian team is composed of young, relative immature skaters; this fact is likely a cause for their inconsistency. Furthermore, Hungary's potential was confirmed at the Olympic Games in February, as they won gold in the relay race; this would lend credence to their increased rank from eighth to third in the novel ranking.

Though this new ranking can be beneficial, it has important drawbacks. Most significantly, the margin of victory used as the foundation of the ranking was employed only if the team in question won the race. That is, if CAN placed second and KOR placed third, the differential between CAN and KOR was excluded from the calculations. Only races in which a country placed first were factored into its margin of victory calculation (hence the name, margin of *victory*).

The other significant drawback of this ranking is that it ignores all other factors in a race other than margin of victory. Races are tremendously complicated and crazy; short track is known as "NASCAR on ice." Thus, important factors that often influence the outcome of a race, such as interference by other teams or differing ice conditions, are not taken into account in this novel ranking.

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