Fractals:
A Visual Display of Mathematics

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Intro:

Fractals are irregular geometric objects that are infinitely complex.

History

How they work

- Self-similarity as a result of affine transformations
- Recursion to compute these transformations

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$

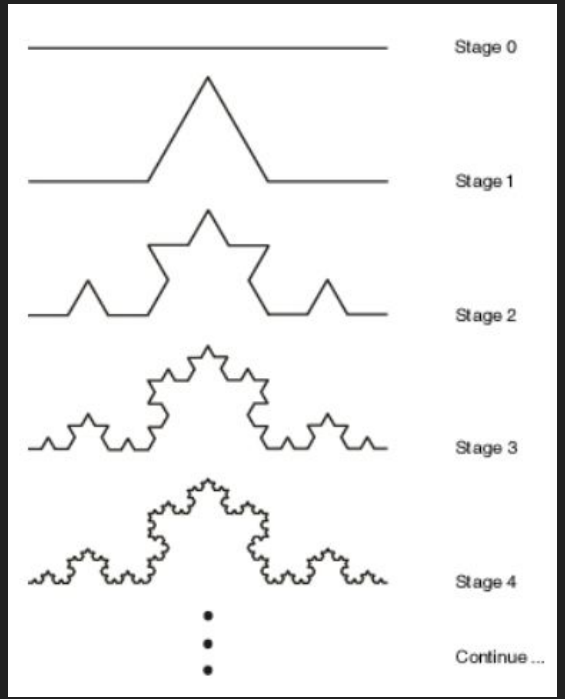
$$T(\mathbf{x}) = A\,\mathbf{x}$$

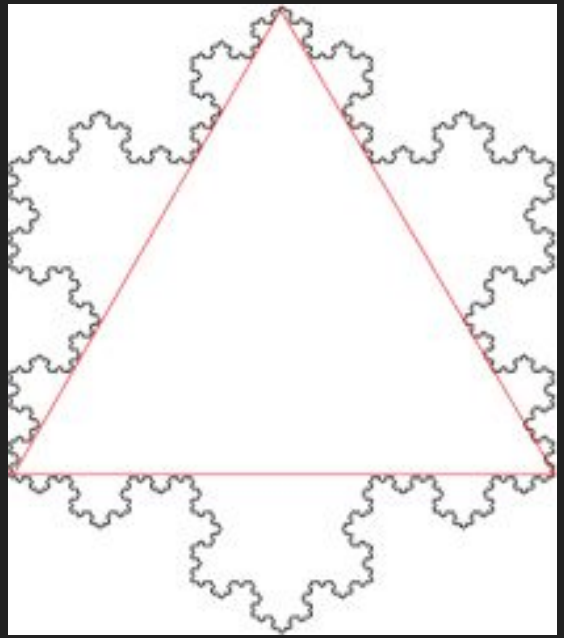
$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

Koch Curve

- Curve gets longer and each section more complex and jagged
- Bounded curve of infinite length
  - Only possible with fractals





Mandelbrot Set

$$Z_n = Z_{n-1}^2 + C$$

- Computed in the complex plane (utilizing imaginary numbers)
  - C = a +bi
- Just a few simple lines of recursive code

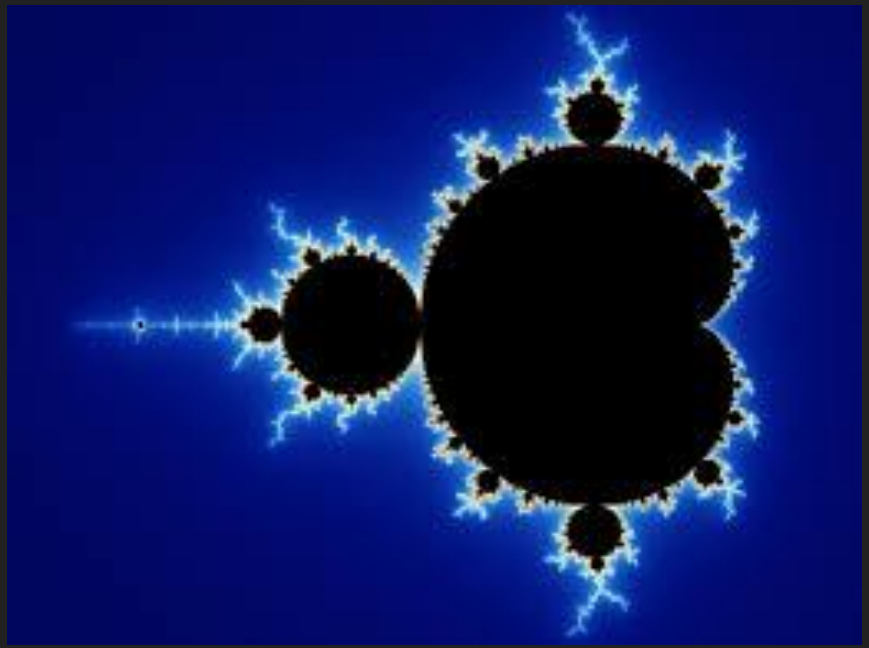
```
mandelbrot := proc(x, y)
local c, z, m;
c := evalf(x+y*I); z := c;
for m to 50 while abs(z) < 2 do z := z^2+c od;
m end;

plots[densityplot](mandelbrot,-2 .. 0.7, -1.35 .. 1.35,
s_tyle=patchnogrid,colorstyle=HUE,numpoints=62500,axes=none);
```

Mandelbrot set

https://youtu.be/0jGaio87u3A

- The Mandelbrot set is the black section in the center of the picture. The color of the pixels outside indicate how many iterations it took for each of those pixels until the distance between that point and the origin exceed the square root of 5, and it is considered outside of the set.



Types of Fractals

1. Sierpinski Triangle
2. Koch
3. Galaxies

Fractals in Nature

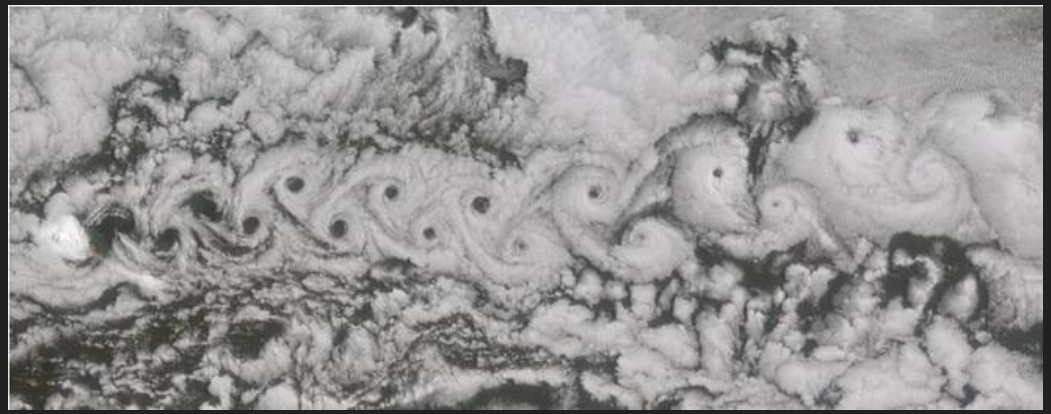
Many geographic features are the result of fractal patterns—mountains, coastlines, rivers, canyons, etc.

To the right is an aerial view of the Himalayas. The movement of the tectonic plates forms the mountains with fractal trends.

Fractals in Nature

Romanesco, the cousin of broccoli, is a great example of fractal patterns in nature. It is a natural representation of the Fibonacci or golden spiral, a logarithmic spiral where every quarter turn is farther from the origin by a factor of phi, the golden ratio

Fractals in Nature



The weather puts on fantastic displays of fractals in cloud, lightning, hurricanes, and more.

Fractals in Nature

The study of fractals opened up a whole new world in regard to the study of the human body. Fractals are everywhere inside of us. Kidneys, blood vessels, lungs, even our brain exhibit fractal formations.