Electrical Circuits

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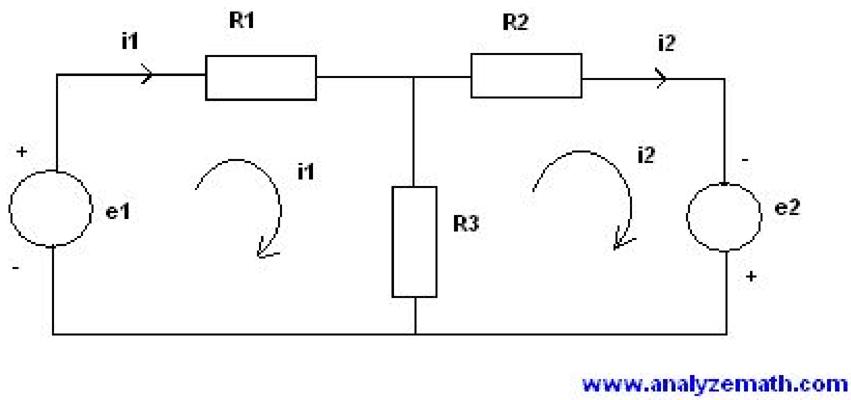
While you can find matrices in most fields of science, they can be used to study the physical phenomenas in mechanics, physics, magnetism, and electrical circuits. Matrices can be described through a system of linear equations, more specifically a two dimensional vector with an input and output. A functioning circuit is built by three main components; a power source, a conductor, and a resistor. Electrons from the voltage source travel along the conductive material and lead to a ground or travel through resistors prior the ground. Electrical circuits can vary from simple to extreme and can take a tremendous amount of time to solve for variables solely using formal laws of physics. Linear functions have one input vector and one output; similarly, electrical circuits travel a path that has a start and finish. To solve for unknown component values of an extensive circuit, a simple matrix can be created and reduced to find all currents, fast and accurately. Using linear algebra, the individual current value for simple circuits can be found and compared to the accurate results found from complex ones using Kirchhoff's Law and Gaussian Elimination.

To determine the electrical current flow in a circuit, a network equation needs to be constructed. Kirchhoff's Current Law states that the total current entering a circuit junction is the same as the total current leaving the same junction. Therefore, the sum of all currents entering and leaving must be equal to zero. Once the network equations are organized, convert the values into a matrix; then, apply Gaussian Elimination (row reduction) to finally have a matrix in the reduced row echelon form. This will provide the individual currents in circuits that vary from simple to complex.



The circuit located above is a simple two loop circuit that can be solved by hand. The power source is e1 and e2, the resistors are R1, R2, and R3, and the current flows are i1 and i2. Kirchhoff's equations are rewritten to suit a matrix.

Kirchhoff's Law $\sum_{k=1}^{n} V_k = 0$

First loop: e1= i1(R1+R3) - i2R3

Second loop: e2= -i1R3 + i2(R2+R3)

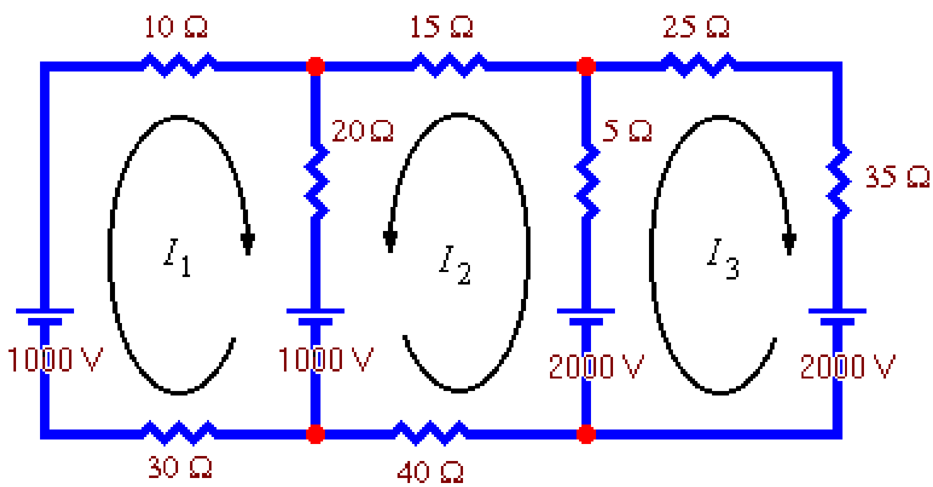
Rewritten in matrix form.

$$\begin{pmatrix} e1 \\ e2 \end{pmatrix} = \begin{pmatrix} R1+R3 & -R3 \\ -R3 & R2+R3 \end{pmatrix} \begin{pmatrix} i1 \\ i2 \end{pmatrix}$$

$$e = \begin{pmatrix} e1 \\ e2 \end{pmatrix} \quad R = \begin{pmatrix} R1+R3 & -R3 \\ -R3 & R2+R3 \end{pmatrix} \quad i = \begin{pmatrix} i1 \\ i2 \end{pmatrix}$$

$$i = R^{-1} e$$

The current flow can be found by calculating the inverse matrix and multiplying by e. A complicated matrix can contain several loops and resistors. Kirchhoff's Law explains that for any closed loop in a circuit, the sum of all voltages on the loop is equal to zero. A matrix with correlating voltages and last column zero, can be manipulated to find the circuit's current. Similar to matrix inversion, reduced row echelon form can also be used to find current values without the understanding of Kirchhoff's laws.



$$10i1 + 20(i1 + i2) + 30i1 = 1000 - 1000$$

$$15i2 + 20(i2 + i1) + 40i2 + 5(i2 + i3) = 2000 - 1000$$

$$25i3 + 35i3 + 5(i3 + i2) = 2000 - 2000$$

Gaussian Elimination:

$$\begin{cases} 60i1 + 20i2 = 0 \\ 20i1 + 80i2 + 5i3 = 1000 \\ 5i2 + 65i3 = 0 \end{cases}$$

$$\begin{bmatrix} 60 & 20 & 0 & 0 \\ 20 & 80 & 5 & 1000 \\ 0 & 5 & 65 & 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc}1 & 1/3 & 0 & 0\\20 & 80 & 5 & 1000\\0 & 5 & 65 & 0\end{array}\right] R1(1/6)$$

$$\left[\begin{array}{cccc}1 & 1/3 & 0 & 0\\0 & 220/3 & 5 & 1000\\0 & 5 & 65 & 0\end{array}\right] -20R1+R2$$

$$\left[\begin{array}{cccc}1 & 1/3 & 0 & 0\\0 & 1 & 3/44 & 150/11\\0 & 5 & 65 & 0\end{array}\right] R2(3/220)$$

$$\left[\begin{array}{cccc}1 & 1/3 & 0 & 0\\0 & 1 & 3/44 & 150/11\\0 & 0 & 2845/44 & -750/11\end{array}\right] -5R2+R3$$

$$\left[\begin{array}{cccc}1 & 1/3 & 0 & 0\\0 & 1 & 3/44 & 150/11\\0 & 0 & 1 & -600/569\end{array}\right] R3(44/2845)$$

$$\left[\begin{array}{cccc}1 & 1/3 & 0 & 0\\0 & 1 & 0 & 7800/569\\0 & 0 & 1 & -600/569\end{array}\right] -3/44R3+R2$$

$$\left[\begin{array}{cccc}1 & 0 & 0 & -2600/569\\0 & 1 & 0 & 7800/569\\0 & 0 & 1 & -600/569\end{array}\right] -1/3R2+R1$$

I1= -4.56   I2= 13.7   I3= -1.05

Here we've solved the current flow in all three loops in seven simple steps by row reduction.

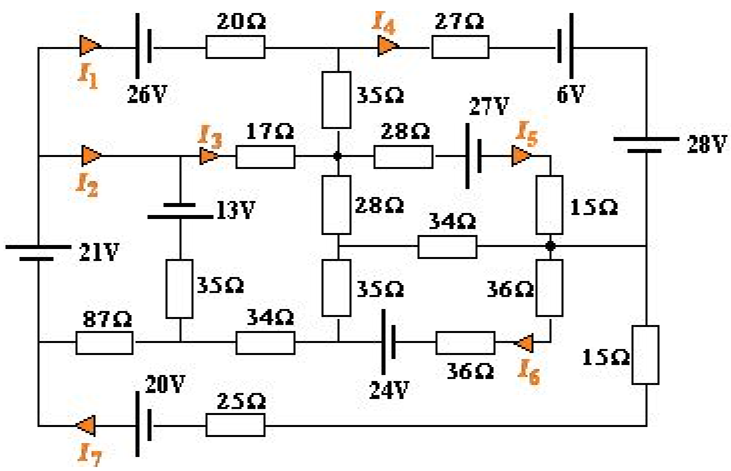
Kirchhoff's Law:

$$\begin{pmatrix}60 & 20 & 0\\20 & 80 & 5\\0 & 5 & 65\end{pmatrix}^{-1} = \begin{pmatrix}\frac{207}{11380} & -\frac{13}{2845} & \frac{1}{2845}\\-\frac{13}{2845} & \frac{39}{2845} & -\frac{3}{2845}\\\frac{1}{2845} & -\frac{3}{2845} & \frac{44}{2845}\end{pmatrix}$$

$$\begin{pmatrix} \frac{207}{11380} & -\frac{13}{2845} & \frac{1}{2845} \\ -\frac{13}{2845} & \frac{39}{2845} & -\frac{3}{2845} \\ \frac{1}{2845} & -\frac{3}{2845} & \frac{44}{2845} \end{pmatrix} \begin{pmatrix} 0 \\ 1000 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{2600}{569} \\ \frac{7800}{569} \\ -\frac{600}{569} \end{pmatrix}$$

I1= -4.56   I2= 13.7   I3= -1.05

The matrix inversion and multiplication were not solved by hand; but as shown above, the values are the same. The next example located below, contains a complex circuit with 7 unknown currents labeled I1 through I7. Similar to the example above, the matrices will be constructed, applied to Kirchoff's law, and compared to Gaussian elimination.



$$-26 = 72I1 - 17I3 - 35I4$$
$$34 = 122I2 - 35I3 - 87I7$$
$$-4 = 233I7 - 87I2 - 34I3 - 72I6$$
$$-13 = 149I3 - 17I1 - 35I2 - 28I5 - 35I6 - 34I7$$
$$-27 = 105I5 - 28I3 - 43I4 - 34I6$$
$$24 = 141I6 - 35I3 - 34I5 - 72I7$$
$$5 = 105I4 - 35I1 - 43I5$$

Kirchhoff's Law:

$$X = \begin{bmatrix} 72 & 0 & -17 & -35 & 0 & 0 & 0 \\ 0 & 122 & -35 & 0 & 0 & 0 & -87 \\ 0 & -87 & -34 & 0 & 0 & -72 & 233 \\ -17 & -35 & 149 & 0 & -28 & -35 & -34 \\ 0 & 0 & -28 & -43 & 105 & -34 & 0 \\ 0 & 0 & -35 & 0 & -34 & 141 & -72 \\ -35 & 0 & 0 & 105 & -43 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -26 \\ 34 \\ -4 \\ -13 \\ -27 \\ 24 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} -0.46801 \\ 0.42932 \\ 5.193 \times 10^{-3} \\ -0.22243 \\ -0.27848 \\ 0.21115 \\ 0.20914 \end{bmatrix}$$

I1= -0.468 I2= 0.429 I3= 0.0005 I4= -0.222 I5= -0.278 I6= 0.211 I7= 0.209

Gaussian Elimination:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -23617908993070/50464671166057 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 21665521993439/50464671166057 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 262065058161/50464671166057 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -11224660090637/50464671166057 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -14053159316040/50464671166057 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 10655498693183/50464671166057 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 10554282603655/50464671166057 \end{bmatrix}$$

$$= \begin{bmatrix} -0.46801 \\ 0.42932 \\ 5.193 \times 10^{-3} \\ -0.22243 \\ -0.27848 \\ 0.21115 \\ 0.20914 \end{bmatrix}$$

$I1= -0.468$ $I2= 0.429$ $I3= 0.0005$ $I4= -0.222$ $I5= -0.278$ $I6= 0.211$ $I7= 0.209$

The results are the same using both methods. Although the reduced row echelon form has larger numbers, the values again are the same.

Using matrices to solve for currents is more efficient than solely using physics formulas. A current can be found by an inverse matrix, row reduction, and numerous other linear algebra applications. You can also find node values, incorporate power switches, multiple batteries, capacitors, and organize the equations to solve for unknown variables. Calculating the inverse matrix using a computer tool was fast and efficient compared to row reduction by hand. The complexity of a circuit can be discouraging but through linear algebra, it can be understood by all audiences.

Resources

ADendane@uaeu.ac.ae. “Free Mathematics Tutorials.” *Matrices Applied to Electric Circuits*, www.analyzemath.com/applied_mathematics/electric_circuit_1.html.

Bourne, Murray. “6. Matrices and Linear Equations.” *Intmathcom RSS*, www.intmath.com/matrices-determinants/6-matrices-linear-equations.php.

Kirchhoff's Voltage Law and the Conservation of Energy. (2018, May 02). Retrieved from https://www.electronics-tutorials.ws/dccircuits/kirchhoffs-voltage-law.html