Quiz 9

Extra Credit Problem 1. Piecewise Continuous Inputs

Consider a passenger SUV on a short trip from Salt Lake City to Evanston, on the Wyoming border. The route is I-80 E, 75 miles through Utah. Google maps estimates 1 hour and 11 minutes driving time. The table below shows the distances, time, road segment and average speed with total trip time 1 hour and 38 minutes. Cities enroute reduce the freeway speed by 10 mph, the trip time effect not shown in the table.

Miles	Minutes	Speed mph	Road Segment	Posted limit mph
18.1	20	54.3	Parley's Walmart to Kimball	65
11.3	12	56.5	Kimball to Wanship	65 – 55
9.1	11	49.6	Wanship to Coalville	70
5.7	6	57	Coalville to Echo Dam	70
16.5	16	61.9	Echo Dam to 75 mph sign	70
39	33	70.9	75 mph sign to Evanston	75

The **velocity function** for the SUV is approximated by

$$V_{\mathrm{pc}}(t) = \begin{cases} \text{Speed mph} & \text{Time interval minutes} & \text{Road segment} \\ \hline 54.3 & 0 < t < 20 & \text{Parley's Walmart to Kimball} \\ 56.5 & 20 < t < 32 & \text{Kimball to Wanship} \\ 49.6 & 32 < t < 43 & \text{Wanship to Coalville} \\ 57.0 & 43 < t < 49 & \text{Coalville to Echo Dam} \\ 61.9 & 49 < t < 65 & \text{Echo Dam to 75 mph sign} \\ 70.1 & 65 < t < 98 & \text{75 mph sign to Evanston} \end{cases}$$

The velocity function $V_{\mathrm{pc}}(t)$ is piecewise continuous, because it has the general form

$$f(t) = \begin{cases} f_1(t) & t_1 < t < t_2 \\ f_2(t) & t_2 < t < t_3 \\ \vdots & \vdots \\ f_n(t) & t_n < t < t_{n+1} \end{cases}$$

where functions $f_1, f_2, \ldots, f_n$ are **continuous on the whole real line** $-\infty < t < \infty$. We don't define $f(t)$ at division points, because of many possible ways to make the definition. As long as these values are not used, then it will make no difference. Both right and left hand limits exist at a division point. For Laplace theory, we like the definition $f(0) = \lim_{h \to 0+} f(h)$, which allows the parts rule $Ł(f'(t)) = sŁ(f(t)) - f(0)$.

The Problem. The SUV travels from $t = 0$ to $t = \frac{98}{60} = 1.6$ hours. The odometer trip meter reading $x(t)$ is in miles (assume $x(0) = 0$). The function $V_{\mathrm{pc}}(t)$ is an approximation to the speedometer reading. Laplace's method can solve the approximation model

$$\frac{dx}{dt} = V_{\mathrm{pc}}(60t), \quad x(0) = 0, \quad x \text{ in miles, } t \text{ in hours,}$$

obtaining $x(t) = \int_0^t V_{\mathrm{pc}}(60w)dw$, the same result as the method of quadrature. Show the details. Then display the piecewise linear continuous trip meter reading $x(t)$.