

Sample Quiz 3 Problem 1

$$u' = u(u^2 - 4)$$

Phase line diagram, $F(u) = u(u^2 - 4) = u(u-2)(u+2)$

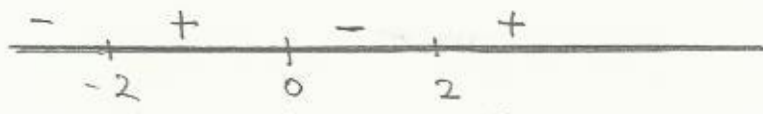
Equilibria are roots of $F(u) = 0$. Then $u = 0, 2, -2$

$$F(-3) = \text{Neg}, F(1) = \text{Neg}$$

$$F(-1) = \text{Pos}, F(3) = \text{Pos}$$

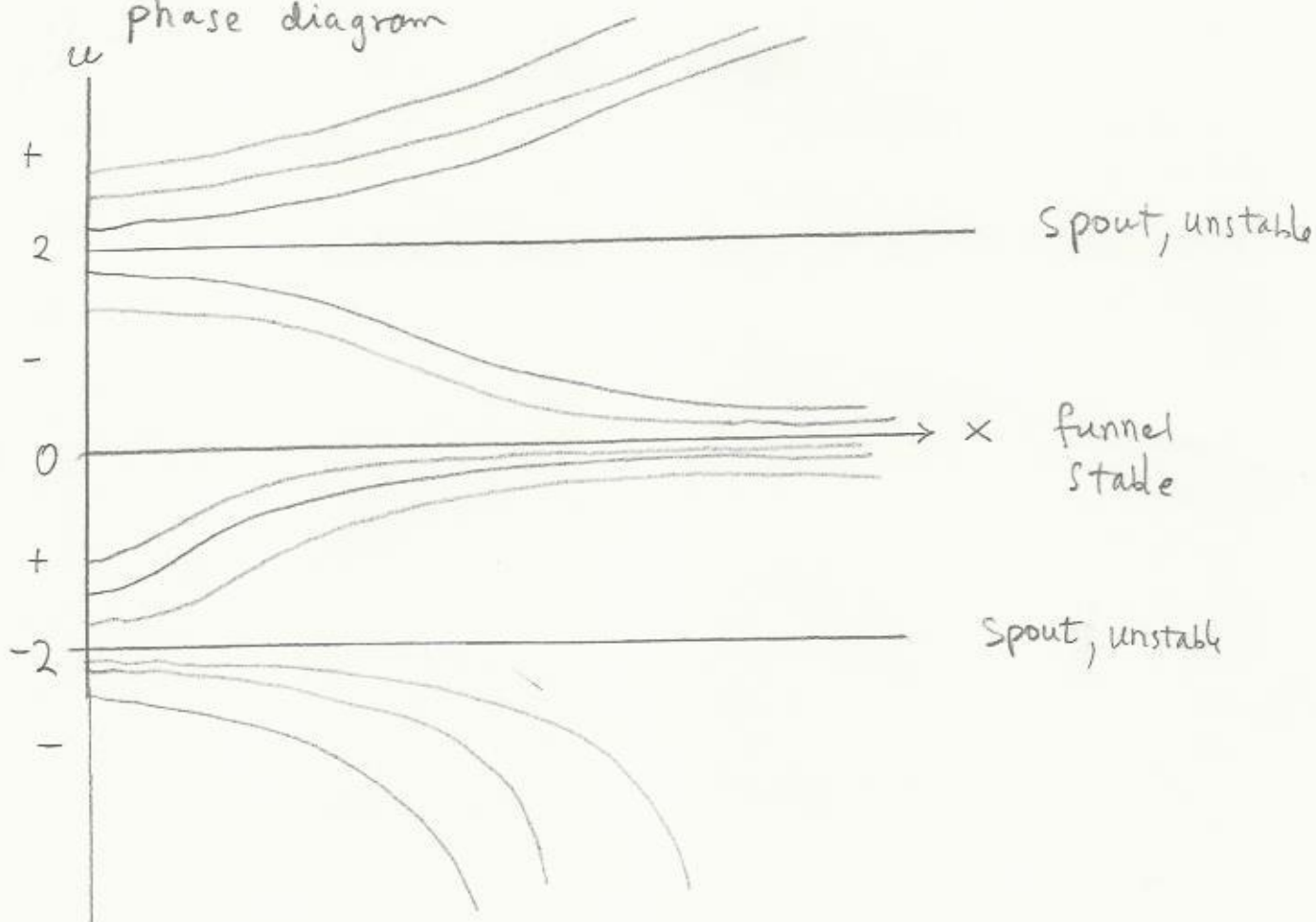
Example: Sign - on $0 < u < 2$

because $F(1) = -3$ (F is one-signed for $0 < u < 2$)



The phase line diagram was constructed from F. we move it to the vertical u-axis, below. Sign + means increasing, - means decreasing.

phase diagram



Theorem

Between 2 adjacent equilibrium points, $F(u)$ is one-signed. Alternate: if $F(a) = F(b) = 0$ and F has no roots in $a < u < b$, then F is one-signed on $a < u < b$.

Sample Quiz 3

Problem 2

$$\begin{cases} y' = \sin(x^2) \\ y(0) = 0 \end{cases}$$

Solve numerically using Rect, Trap, Simp

RECT The approx formula is $y(x_0+h) = y(x_0) + h F(x_0)$, because $\int_a^b F dx \approx F(a)(b-a)$ for $b-a$ small. Then the supplied table implies

$$\begin{aligned} y(0.6) &= y(0.4) + \int_{0.4}^{0.6} F dx \approx 0.007997866838 + (0.2) F(0.4) \\ &= 0.007997866838 + (0.2) \sin(0.4^2) \\ &= 0.03986150816 \end{aligned}$$

$$\begin{aligned} y(0.8) &= 0.03986150816 + 0.2 \sin(0.6^2) \\ &= 0.1103163548 \end{aligned}$$

TRAP The approx formula $\int_a^b F dx \approx (b-a) \frac{F(a)+F(b)}{2}$ implies that $y(x_0+h) = y(x_0) + \frac{h}{2} (F(x_0) + F(x_0+h))$. The Table implies

$$\begin{aligned} y(0.2) &= y(0.0) + \frac{0.2}{2} (F(0) + F(0.2)) \\ &= 0 + 0.1 (\sin(0) + \sin(0.2^2)) \\ &= 0.003998933419 \end{aligned}$$

$$\begin{aligned} y(0.8) &= y(0.6) + 0.1 (F(0.6) + F(0.8)) \\ &= 0.07508893150 + 0.1 (\sin(0.6^2) + \sin(0.8^2)) \\ &= 0.1700358989 \end{aligned}$$

RK4 Use $\int_a^b F dx \approx \frac{(b-a)}{6} (F(a) + 4F(\frac{a+b}{2}) + F(b))$ to get

$y(x_0+h) = y(x_0) + \frac{h}{6} (F(x_0) + 4F(x_0+h/2) + F(x_0+h))$. The Table implies

$$\begin{aligned} y(0.6) &= y(0.4) + \frac{0.2}{6} (F(0.4) + 4F(0.5) + F(0.6)) \\ &= 0.2129368017 + \frac{0.1}{3} (\sin(0.4^2) + 4\sin(0.5^2) + \sin(0.6^2)) \\ &= 0.07133395608 \end{aligned}$$

$$\begin{aligned} y(1.0) &= y(0.8) + \frac{0.1}{6} (\sin(0.8^2) + 4\sin(0.9^2) + \sin(1.0^2)) \\ &= 0.3102602343 \end{aligned}$$

Solution Key

0.0039989334

0.039861508

0.110316354

0.1700358989

0.07133395608

0.3102602343