Name ________________________________

Differential Equations 2280

Midterm Exam 2

Exam Date: 3 April 2015 at 12:50pm

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

1. **(Chapter 3)**

(a) [70%] Find the steady-state periodic solution for the spring-mass equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 85\cos(t).$$

(b) [30%] Solve for the general solution of the homogeneous constant-coefficient differential equation whose characteristic equation is

$$r^2(r^2 - r)^2(r^2 + 2r + 5)^2 = 0.$$

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2. (Chapters 1, 2, 3)

(a) [40%] Find the factors of the characteristic equation of a linear homogeneous constant coefficient differential equation of lowest order which has a particular solution

$$y(x) = 10 + 4\cos(2x) + 5xe^{x}\sin(x).$$

(b) [60%] Determine for differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + xe^{-x}$$

the shortest trial solution for y_p according to the method of undetermined coefficients. Do not evaluate the undetermined coefficients!

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3. (Laplace Theory)

(a) [60%] Assume $f(t)$ is of exponential order. Find $f(t)$ in the relation

$$\left.\left(\frac{d^2}{ds^2}\mathcal{L}(f(t))\right)\right|_{s\to(s+4)} = \mathcal{L}(f(t)) + \frac{s^2+2}{s^3+s}.$$

(b) [40%] Find $\mathcal{L}(f)$ given $f(t) = e^{2t}\sin(3t) + (t+1)^2 e^t$.

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4. **(Laplace Theory)**

(a) [40%] The solution of $x''' + x' = 0$, $x(0) = 1$, $x'(0) = 0$, $x''(0) = 0$ is $x(t) = 1$. Show the details in Laplace's Method for obtaining this answer.

(b) [60%] Solve the system $x' = x - y$, $y' = x + y + 2$, $x(0) = 0$, $y(0) = 0$ by Laplace's Method.

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5. (Laplace Theory)
Compute $\mathcal{L}\left(\dfrac{\sinh(2t)}{t}\right)$

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6. (Systems of Differential Equations)

Let $A = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$. The eigenvalues of A are 4, 6.

(a) [30%] Find all entries of the 2×2 exponential matrix e^{At} according to Putzer's spectral formula.

(b) [40%] Display the solution of $\mathbf{u}' = A\mathbf{u}$, $\vec{u}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, according to the Cayley-Hamilton-Ziebur shortcut. The scalar form of the system is

$$\begin{cases} x'(t) &= 5x(t) + y(t), \\ y'(t) &= x(t) + 5y(t), \\ x(0) &= 1, \\ y(0) &= -1. \end{cases}$$

(c) [30%] The eigenpairs of a 3×3 matrix C are

$$\left(0, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right), \quad \left(1, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right), \quad \left(2, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}\right).$$

Display the general solution of $\mathbf{u}' = C\mathbf{u}$ by the eigenanalysis method.

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