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Review Problem 4 (Chapter 7)

a) $x'' + x = \cos t$, $x(0) = x'(0) = 0$

$$\mathcal{L}(x'' + x) = \mathcal{L}(\cos t)$$

$$\mathcal{L}(x'') + \mathcal{L}(x) = \mathcal{L}(\cos t)$$

$$s^2 \mathcal{L}(x) + \mathcal{L}(x) = \mathcal{L}(\cos t)$$

$$(s^2+1)\mathcal{L}(x) = \mathcal{L}(\cos t)$$

$$\mathcal{L}(x) = \frac{\mathcal{L}(\cos t)}{s^2+1}$$

$$\mathcal{L}(x) = \frac{\frac{s}{s^2+1}}{s^2+1}$$

$$\mathcal{L}(x) = \frac{s}{(s^2+1)^2}$$

$$\frac{d}{ds}\left(\frac{1}{s^2+1}\right) = \frac{-2s}{(s^2+1)^2}$$

$$\frac{s}{(s^2+1)^2} = -\frac{1}{2}\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$$

$$\mathcal{L}(x) = -\frac{1}{2}\frac{d}{ds}\mathcal{L}(\sin t)$$

$$\mathcal{L}(x) = \frac{1}{2}\mathcal{L}(t\sin t)$$

$$\cancel{\mathcal{L}}(x) = \cancel{\mathcal{L}}\left(\frac{1}{2}t\sin t\right)$$

$$x = \frac{1}{2}t\sin t$$

b) $\lim\limits_{s\to\infty} \mathcal{L}(f(t)) = 0$

$\lim\limits_{s\to\infty} \frac{s}{s+1} = 1 \neq 0 \Rightarrow$ Does Not Exist

c) Linearity: $\mathcal{L}(cf) = c\mathcal{L}(f)$

Shift: $\mathcal{L}(e^{at}f(t)) = \mathcal{L}(f(t))\big|_{s \to s-a}$

S-differentiation: $\mathcal{L}((-t)f(t)) = \frac{d}{ds}\mathcal{L}(f(t))$

Lerch: $\mathcal{L}(f) = \mathcal{L}(g) \Rightarrow f = g$

Parts: $\mathcal{L}(f'(t)) = -f(0) + s\mathcal{L}(f(t))$

Periodic: $\mathcal{L}(f(t)) = \dfrac{\int_0^p f(t) e^{-st} dt}{1 - e^{-ps}}$, $f(t) = f(t+p)$

Convolution: $\mathcal{L}(f_1)\mathcal{L}(f_2) = \mathcal{L}\left(\int_0^t f_1(x) f_2(t-x)\, dx\right)$

d) We haven't covered resolvent theory in class. This method won't be on the test.

e) $y''(t) + y(t) = f(t)$, $y(0) = y'(0) = 0$

$\mathcal{L}(y'' + y) = \mathcal{L}(f)$

$\mathcal{L}(y'') + \mathcal{L}(y) = \mathcal{L}(f)$

$s^2 \mathcal{L}(y) + \mathcal{L}(y) = \mathcal{L}(f)$

$(s^2+1)\mathcal{L}(y) = \mathcal{L}(f)$

$\mathcal{L}(y) = \dfrac{1}{s^2+1} \mathcal{L}(f)$

$\mathcal{L}(y) = \mathcal{L}(\sin t)\mathcal{L}(f(t))$

By convolution:

$y(t) = \int_0^t \sin(t-x) f(x)\, dx$