

1) a) $y'' + 4y' + 4y = 0$

$$r^2 + 4r + 4 = 0$$

$$(r+2)^2 = 0$$

$$r = -2, -2$$

$$e^{-2x}, xe^{-2x}$$

$$y = c_1 e^{-2x} + c_2 x e^{-2x}$$

b) $y^{(5)} + 4y^{(4)} = 0$

$$r^5 + 4r^4 = 0$$

$$r^4(r+4) = 0$$

$$r = 0, 0, 0, 0, -4$$

$$1, x, x^2, x^3, e^{-4x}$$

$$y = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + c_5 e^{-4x}$$

c) $r(r-3)(r^3-9r)^2(r^2+4)^3 = 0$

$$r(r-3)(r^3-9r)^2(r^2+4)^3 = 0$$

$$r = 0, 0, 0, 3, 3, 3, -3, -3, \pm 2i, \pm 2i, \pm 2i$$

$$1, x, x^2, e^{3x}, xe^{3x}, x^2 e^{3x}, e^{-3x}, xe^{-3x}$$

$$\cos 2x, \sin 2x, x \cos 2x, x \sin 2x, x^2 \cos 2x, x^2 \sin 2x$$

$$y = c_1 + c_2 x + c_3 x^2 + c_4 e^{3x} + c_5 x e^{3x} + c_6 x^2 e^{3x}$$

$$+ c_7 e^{-3x} + c_8 x e^{-3x} + (c_9 + c_{10} x + c_{11} x^2) \cos 2x + (c_{12} + c_{13} x + c_{14} x^2) \sin 2x$$

d) $6x''(t) + 7x'(t) + 2x(t) = 0$

$$6r^2 + 7r + 2 = 0$$

$$(3r+2)(2r+1) = 0$$

$$r = -\frac{2}{3}, -\frac{1}{2}$$

$$y = c_1 e^{-2/3x} + c_2 e^{-1/2x}$$

overdamped

